T-79.1001 **Autumn 2005**
Introduction to Theoretical Computer Science T
Tutorial 9, 22–23 November
Problems

Homework problems:

1. Design a Turing machine that recognises ("decides") the language

$$\{1^n 0 1^n \mid n \geq 0\}.$$

Show the computation sequences ("runs") of your machine on inputs 11011 and 1011.

2. Design a two-tape nondeterministic Turing machine that recognises the language $\{ww \mid w \in \{a, b\}^*\}$.

3. Design a three-tape Turing machine ADD that functions as follows. The machine gets as input on tapes 1 and 2 two binary numbers written in reverse, i.e. with their least significant bits first. It then computes on tape 3 the sum of the two given numbers in the same notation. For simplicity, you may assume that the input numbers are of the same length, i.e. that the possibly shorter one is padded with leading zeros. Thus, for instance, the calculation $7 + 11 = 18$ is represented as:

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1110
1101
01001
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Demonstration problems:

4. Show that pushdown automata with *two* stacks (rather than just one as permitted by the standard definition) would be capable of recognising exactly the same languages as Turing machines.

5. Extend the notion of a Turing machine by providing the possibility of a *two-way infinite tape*. Show that nevertheless such machines recognise exactly the same languages as the standard machines whose tape is only one-way infinite.

6. Show that Turing machines whose tape alphabet contains at most two symbols in addition to the input symbols are capable of recognising exactly the same languages as the standard machines.