T–79.1001 **Autumn 2006**
Introduction to Theoretical Computer Science T
Tutorial 9, 21 to 22 November
Problems

Homework problems:

1. Design a two-tape nondeterministic Turing machine that recognises the language $\{ww \mid w \in \{a, b\}^*\}$. You may assume that the tape head of the machine can also stay stationary in a transition (move direction "S").

2. Design a three-tape Turing machine ADD that functions as follows. The machine gets as input on tapes 1 and 2 two binary numbers written in reverse, i.e. with their least significant bits first. It then computes on tape 3 the sum of the two given numbers in the same notation. For simplicity, you may assume that the input numbers are of the same length, i.e. that the possibly shorter one is padded with leading zeros. Thus, for instance, the calculation 7 + 11 = 18 is represented as:

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1110
1101
01001
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3. Let A and B be countably infinite sets such that $A \cap B = \emptyset$. Show that then also the set $A \cup B$ is countably infinite. (*Extra question:* Show that the claim holds even without the assumption $A \cap B = \emptyset$.)

Demonstration problems:

4. Show that pushdown automata with *two* stacks (rather than just one as permitted by the standard definition) would be capable of recognising exactly the same languages as Turing machines.

5. Extend the notion of a Turing machine by providing the possibility of a *two-way infinite tape*. Show that nevertheless such machines recognise exactly the same languages as the standard machines whose tape is only one-way infinite.

6. Prove that the Cartesian product $\mathbb{N} \times \mathbb{N}$ is countably infinite. (*Hint:* Think of the pairs $(m, n) \in \mathbb{N} \times \mathbb{N}$ as embedded in the Euclidean (x, y) plane $\mathbb{R}^2$. Enumerate the pairs by diagonals parallel to the line $y = -x$.) Conclude from this result that also the set $\mathbb{Q}$ of rational numbers is countably infinite.