

Helsinki University of Technology
Laboratory for Theoretical Computer Science

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T-79.1001 Introduction to Theoretical Computer Science T (4 ECTS)

Exam Thu 9 Mar 2006, 4–7 p.m.

Write down on each answer sheet:

- Your name, department, and student id
 - The text: “T-79.1001 Introduction to Theoretical Computer Science T 9.3.2006”
 - The total number of answer sheets you are submitting for grading
- This exam corresponds to the pre-2005 course T-79.148.

1. Show that each of the following languages is regular, by describing it either in terms of a regular expression or in terms of a finite automaton:

- (a) $\{w \in \{0, 1\}^* \mid \text{each two 1's in } w \text{ are separated by an even number of 0's (possibly none)}\}$,
5p.
- (b) $\{w \in \{0, 1\}^* \mid w \text{ contains substring } 11 \text{ exactly once}\}$,
5p.
- (c) $\{w \in \{0, 1\}^* \mid w \text{ does not contain substring } 111\}$.
5p.

2. (a) Show that the following context-free grammar is ambiguous:

$$S \rightarrow aSb \mid aSbb \mid \epsilon.$$

5 p.

- (b) Design an unambiguous grammar generating the same language as the grammar in part (a).
5 p.
- (c) Prove (precisely!) that the language generated by the grammars in parts (a) and (b) is not regular.
5 p.

3. Design a deterministic single-tape Turing machine that duplicates its input: if the tape initially contains a string $w \in \{0, 1\}^*$, then when the machine halts the tape contains the string ww . Present your Turing machine as a state diagram, and give its computation sequence on input 10.
15p.

4. *One* of the following:

- (a) Explain how you would determine (systematically) whether the language described by a regular expression r over the alphabet $\{0, 1\}$ is (a) empty, i.e. $L(r) = \emptyset$, (b) contains all possible binary strings, i.e. $L(r) = \{0, 1\}^*$.
15p.
- (b) Assume that you are explaining the key contents of the course “Introduction to Theoretical Computer Science T” to a friend who has not yet taken the course. Describe the Church-Turing thesis to her, and convince her of the fact that there are problems that cannot be solved by a computer.
15p.

Total 60p.