Helsinki University of Technology
Laboratory for Theoretical Computer Science
Pekka Orponen (tel. 5246), Tommi Syrjänen (tel. 5082)

T-79.1001 Introduction to Theoretical Computer Science T (4 ECTS)
Exam Thu 19 May 2006, 3–5 p.m.

Write down on each answer sheet:
- Your name, department, and student id
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 19.5.2006"
- The total number of answer sheets you are submitting for grading

This exam corresponds to the pre-2005 course T-79.148.

1. Show that each of the following languages is regular, by describing it either in terms of a regular expression or in terms of a finite automaton:

   (a) $\{w \in \{0,1\}^* \mid w$ contains three consequent zeros or three consequent ones (or both)$\}$; *4p.*

   (b) $\{w \in \{0,1\}^* \mid w$ contains neither three consequent zeros nor three consequent ones$\}$; *4p.*

   (c) $\{w \in \{0,1\}^* \mid$ the number of ones in w is a multiple of three (possibly zero)$\}$; *4p.*

   (d) $\{w \in \{0,1\}^* \mid |w| \geq 3$ and the third-to-last symbol in w is a $1\}$. *4p.*

2. (a) Show that the following context-free grammar is ambiguous:

$$S \to aSb \mid A$$
$$A \to abA \mid \varepsilon$$

   4 p.

   (b) Design an unambiguous grammar generating the same language as the grammar in part (a). *5 p.*

   (c) Prove (precisely!) that the language generated by the grammars in parts (a) and (b) is not regular. *5 p.*

3. Design a deterministic single-tape Turing machine that recognises ("decides") the language

$$L = \{0^n 1 0^n \mid n \geq 0\}.$$

   Present your Turing machine as a state diagram, and give its computation sequences on inputs 010 and 0101. *15p.*

4. (a) Design a (nondeterministic) pushdown automaton that recognises (accepts) the language L considered in the previous problem. (Present the automaton preferably as a state diagram rather than a transition table.) Show the accepting computation sequence ("run") of your automaton on input 010. *5p.*

   (b) Show that also the complement of the language L considered above, i.e. the language

$$\bar{L} = \{x \in \{0,1\}^* \mid x \notin L\},$$

   is context-free, and hence can be recognised by a nondet. pushdown automaton. *10p.*

Total 60p.