**Tik-79.148** **Spring 2001**
**Introduction to Theoretical Computer Science**
**Tutorial 7**
**Exercises**

**Ordinary exercises:**

1. Give a pushdown automaton that accepts the language generated by the context-free grammar $G = (V, \Sigma, R, S)$, where

$$\begin{aligned} V &= \{S, (, ), ^*, \cup, \emptyset, a, b\} \\ \Sigma &= \{(, ), ^*, \cup, \emptyset, a, b\} \\ R &= \{S \to (SS), S \to S^*, S \to (S \cup S), \\ &\quad\; S \to \emptyset, S \to a, S \to b\} \end{aligned}$$

2. Let $L = \big\{w \in \{2,3,4\}^* \mid w = 2^m(3 \cup 4)^n \text{ for some } m \geq 1,\, n \geq m\}\big\}$. Give a pushdown automaton that accepts the language

$$L' = (0 \cup 1 \cup L)^* \quad (\subseteq \{0,1,2,3,4\}^*)$$

   a) using a "direct" construction,
   b) by first defining a context-free grammar that generates the language $L'$ and then constructing a pushdown automaton that corresponds to the grammar.

3. Show that the language $\{a^m b^n c^p d^q \mid n = q \text{ or } m \leq p \text{ or } m+n = p+q\}$ is context-free. Hint: The union of two context-free languages is always context-free.

**Demonstration exercises:**

1. Construct pushdown automata $M = (K, \Sigma, \Gamma, \Delta, s, F)$ that accept the languages

   a) $\{a^m b^n \mid m \leq n \leq 2m\}$
   b) $\{w \in \{a,b\}^* \mid w = w^R\}$

2. Let $M = (K, \Sigma, \Gamma, \Delta, s, F)$. Let $L_f(M)$ be the language defined as follows:

$$L_f(M) = \{w \in \Sigma^* \mid (s, w, e) \vdash_M^* (f, e, \alpha) \text{ for some } f \in F, \alpha \in \Gamma^*\}$$

   a) Show that there exists a pushdown automaton $M'$ such that $L(M') = L_f(M)$.

b) Show that there exists a pushdown automaton $M''$ such that $L_f(M'') = L(M)$.

3. Give a grammar that corresponds to the language accepted by the pushdown automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$, where

$$
\begin{aligned}
K =& \{s, q, f\} \\
\Sigma =& \{a, b\} \\
\Gamma =& \{a, b, c\} \\
F =& \{f\} \\
\Delta =& \big\{\big((s, e, e), (q, c)\big), \big((q, a, c), (q, ac)\big), \big((q, a, a), (q, aa)\big) \\
& \big((q, a, b), (q, e)\big), \big((q, b, c), (q, bc)\big), \big((q, b, b), (q, bb)\big) \\
& \big((q, b, a), (q, e)\big), \big((q, e, c), (f, e)\big)\big\}
\end{aligned}
$$