T-79.148 **Autumn 2002**
Introduction to Theoretical Computer Science
Tutorial 5, 16–18 October
Problems

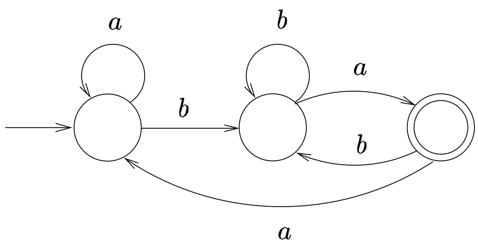
Homework problems:

1. Give regular expressions describing the following languages:
   (a) $\{w \in \{a,b\}^* \mid w$ contains exactly two a's$\}$
   (b) $\{w \in \{a,b\}^* \mid w$ contains at least two a's$\}$
   (c) $\{w \in \{a,b\}^* \mid w$ contains an even number of a's$\}$
   (d) $\{w \in \{a,b\}^* \mid w$ contains either aa or bb (or both) as a substring$\}$
   (e) $\{w \in \{a,b\}^* \mid w$ contains neither aa nor bb as a substring$\}$
   (f) $\{w \in \{0,1\}^* \mid$ the third-to-last symbol in w is $1\}$.
   (g) $\{w \in \{a,\ldots,z,0,\ldots,9,.,@\}^* \mid w$ is a valid e-mail address$\}$;
   (h) $\{w \in \{a,\ldots,z,0,\ldots,9,.,@\}^* \mid w$ is a valid e-mail address ending in the country code '.fi' for Finland$\}$.

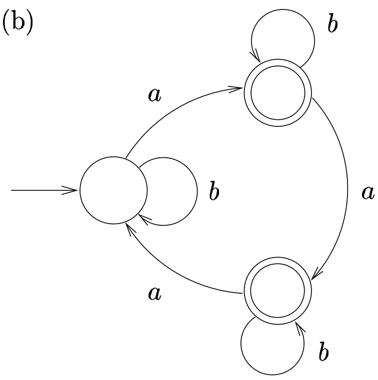
2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression $(a \cup b)^* a (a \cup b)$.
   (b) Make your automaton deterministic.

3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:

(a)



(b)



Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) $(\emptyset^* \cup a)(a^*)^*(b \cup a)b^*$
   (b) $(a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*$
   (c) $a(b^* \cup a^*)(a^*b^*)^*$

5. Determine whether the regular expressions $r_1 = b^*a(a^*b^*)^*$ and $r_2 = (a \cup b)^*a(a \cup b)^*$ describe the same language, by constructing the minimal deterministic finite automata corresponding to them.

6. Prove that if L is a regular language, then so is $L' = \{xy \mid x \in L, y \notin L\}$.