T-79.148 **Spring 2002**
Introduction to Theoretical Computer Science
Tutorial 8, 19–21 March
Problems

Homework problems:

1. Convert the grammar

$$\begin{aligned} S &\rightarrow (S) \mid A \\ A &\rightarrow SS \mid \varepsilon \end{aligned}$$

into Chomsky normal form.

2. Determine, using the CYK algorithm ("dynamic programming method", Lewis & Papadimitriou p. 155), whether the strings $aaaaa$ and $aaaaaa$ are generated by the grammar

$$\begin{aligned} S &\rightarrow AB \mid BC \\ A &\rightarrow BA \mid a \\ B &\rightarrow CC \mid b \\ C &\rightarrow AB \mid a \end{aligned}$$

In the positive case, give also the respective parse tree(s).

3. Design pushdown automata recognising the following languages:

   (a) $\{wcw^R \mid w \in \{a,b\}^*\}$;
   (b) $\{ww^R \mid w \in \{a,b\}^*\}$.

Demonstration problems:

4. Given a context-free grammar $G = (V, \Sigma, P, S)$, a nonterminal $A \in V - \Sigma$ is *redundant*, if it cannot appear in the derivation of any sentence generated by G, i.e. if no derivation in G is of the form $S \Rightarrow^* \alpha A \beta \Rightarrow^* x$, where $\alpha, \beta \in V^*$, $x \in \Sigma^*$. Design an algorithm for removing all the redundant nonterminals from a grammar. (*Hint:* Determine first the *non*redundant nonterminals.)

5. Design a pushdown automaton corresponding to the grammar $G = (V, \Sigma, P, S)$, where

$$\begin{aligned} V &= \{S, (,), {}^*, \cup, \emptyset, a, b\} \\ \Sigma &= \{(,), {}^*, \cup, \emptyset, a, b\} \\ P &= \{S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), \\ &\quad\; S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b\} \end{aligned}$$

6. Design a grammar corresponding to the pushdown automaton $M = (Q, \Sigma, \Gamma, \Delta, s, F)$, where

$$\begin{aligned} Q =& \{s, q, f\},\ \Sigma = \{a, b\},\ \Gamma = \{a, b, c\},\ F = \{f\}, \\ \Delta =& \big\{ \big((s, e, e), (q, c)\big), \big((q, a, c), (q, ac)\big), \big((q, a, a), (q, aa)\big) \\ & \big((q, a, b), (q, e)\big), \big((q, b, c), (q, bc)\big), \big((q, b, b), (q, bb)\big) \\ & \big((q, b, a), (q, e)\big), \big((q, e, c), (f, e)\big) \big\} \end{aligned}$$