**T-79.148** **Spring 2003**
**Introduction to Theoretical Computer Science**
**Tutorial 10, 24–26 March**
**Problems**

**Homework problems:**

1. Design a two-tape Turing machine that recognises the language $\{wcw \mid w \in \{a, b\}^*\}$. Construct the machine so that it makes use of both of its tapes. For simplicity, you may assume that one of the tape heads can stay stationary (direction code "$S$") while the other one moves.

2. Design a three-tape Turing machine ADD that functions as follows. The machine gets as input on tapes 1 and 2 two binary numbers written in reverse, i.e. with their least significant bits first. It then computes on tape 3 the sum of the two given numbers in the same notation. For simplicity, you may assume that the input numbers are of the same length, i.e. that the possibly shorter one is padded with leading zeros. Thus, for instance, the calculation $7 + 11 = 18$ is represented as:

   ```
   1110
   1101
   01001
   ```

3. Design a nondeterministic Turing machine that recognises the language $\{ww \mid w \in \{a, b\}^*\}$. How would you recognise the same language using a deterministic machine?

**Demonstration problems:**

4. Extend the notion of a Turing machine by providing the possibility of a *two-way infinite tape*. Show that nevertheless such machines recognise exactly the same languages as the standard machines whose tape is only one-way infinite.

5. Show that Turing machines whose tape alphabet contains at most two symbols in addition to the input symbols are capable of recognising exactly the same languages as the standard machines.