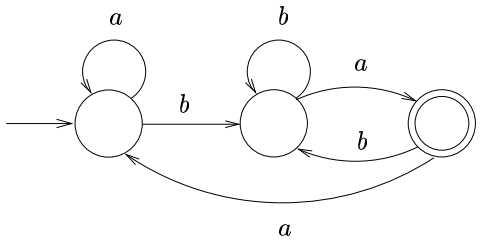
T-79.148 **Spring 2003**
Introduction to Theoretical Computer Science
Tutorial 5, 17 – 19 February
Problems

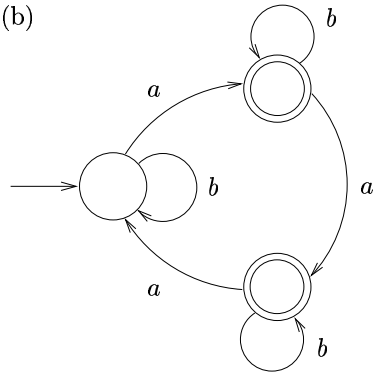
Homework problems:

1. Give regular expressions describing the following languages:
   (a) $\{w \in \{a,b\}^* \mid w$ contains exactly two a's$\}$
   (b) $\{w \in \{a,b\}^* \mid w$ contains at least two a's$\}$
   (c) $\{w \in \{a,b\}^* \mid w$ contains an even number of a's$\}$
   (d) $\{w \in \{a,b\}^* \mid w$ contains either aa or bb (or both) as a substring$\}$
   (e) $\{w \in \{a,b\}^* \mid w$ contains neither aa nor bb as a substring$\}$
   (f) $\{w \in \{0,1\}^* \mid w$ begins and ends with different symbols$\}$;
   (g) $\{w \in \{0,1\}^* \mid |w| = 1 \pmod 3\}$;
   (h) $\{w \in \{a,\ldots,z,0,\ldots,9,.,@\}^* \mid w$ is a valid e-mail address$\}$;
   (i) $\{w \in \{a,\ldots,z,0,\ldots,9,.,@\}^* \mid w$ is a valid e-mail address ending in the country code '.fi' for Finland$\}$.
2. (a) Construct in a systematic way (as described in your textbook) a nondeterministic finite automaton corresponding to the regular expression $((\varepsilon \cup 1)0)^*11^*$.
   (b) Make your automaton deterministic.
   (c) Describe the language in part (a) in English as simply as you can.
3. Construct in a systematic way (as described in your textbook) regular expressions corresponding to the following finite automata:

(a)



(b)



Demonstration problems:

4. Simplify the following regular expressions (i.e., design simpler expressions describing the same languages):
   (a) $(\emptyset^* \cup a)(a^*)^*(b \cup a)b^*$
   (b) $(a \cup b)^* \cup \emptyset \cup (a \cup b)b^*a^*$
   (c) $a(b^* \cup a^*)(a^*b^*)^*$
5. Determine whether the regular expressions $r_1 = b^*a(a^*b^*)^*$ and $r_2 = (a \cup b)^*a(a \cup b)^*$ describe the same language, by constructing the minimal deterministic finite automata corresponding to them.
6. Prove that if L is a regular language, then so is $L' = \{xy \mid x \in L, y \notin L\}$.