T-79.148 **Autumn 2004**
Introduction to Theoretical Computer Science
Tutorial 7, 2–3 November
Problems

Homework problems:

1. Design a right-linear grammar that generates the language

$$\{w \in \{a,b\}^* \mid w \text{ does not contain } abb \text{ as a substring}\}.$$

(Cf. Tutorial 3, Problem 1c.)

2. Consider the following grammar generating a certain type of list structures:

$$S \to (S) \mid S, S \mid a.$$

   (a) Based on the above grammar, give a leftmost and rightmost derivation and a parse tree for the sentence "$(a, (a))$".
   (b) Prove that the grammar is ambiguous.
   (c) Design an unambiguous grammar generating the same language.

3. Show that the language

$$\{0^n 1^m 0^{n-m} \mid n \geq m \geq 0\}$$

is context-free, but not regular.

Demonstration problems:

4. Prove that the class of context-free languages is closed under unions, concatenations, and the Kleene star operation, i.e. if the languages $L_1, L_2 \subseteq \Sigma^*$ are context-free, then so are the languages $L_1 \cup L_2$, $L_1 L_2$ and L_1^*.

5. (a) Prove that the following context-free grammar is ambiguous:

$$\begin{aligned} S &\to \textbf{if } b \textbf{ then } S \\ S &\to \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\to s. \end{aligned}$$

   (b) Design an unambiguous grammar that is equivalent to the grammar in item (a), i.e. that generates the same language. (*Hint:* Introduce new nonterminals B and U that generate, respectively, only "balanced" and "unbalanced" **if-then-else-**sequences.)

6. Design a recursive-descent (top-down) parser for the grammar from Problem 6/6.