T-79.148 **Spring 2004**

Introduction to Theoretical Computer Science

Tutorial 3

Problems

Homework problems:

1. Design finite automata that recognise the following languages:
   (a) $\{w \in \{a,b\}^* \mid w \text{ contains } abab \text{ as a substring}\}$;
   (b) $\{w \in \{a,b\}^* \mid w \text{ does not contain } bba \text{ as a substring}\}$;
   (c) $\{w \in \{a,b\}^* \mid 00 \text{ occurs at most twice as a substring in } w\}$;
   (d) $\{w \in \{0,1\}^* \mid \text{the number of 1's in } w \text{ is divisible by three}\}$;

2. Design a finite automaton that accepts precisely those binary strings where the number of 0's is even and the number of 1's is divisible by three (e.g. 00111, 10101 and 00, but not 1010 or 1011). [NB. In this and similar problems in the future, it is for simplicity always assumed that also zero is even, divisible by three, etc., unless otherwise indicated.]

3. Design a finite automaton (state machine) that models the behaviour of a simple TV set. The channel selector of the TV has three positions (1/2/3), and the voice control has two (lo/hi). The TV is assumed to be always on, so the automaton does not need to have any distinct start or final states.

Demonstration problems:

4. Formulate the model of a simple coffee machine presented in class (lecture notes p. 17) precisely according to the mathematical definition of a finite automaton (Definition 2.1). What is the formal language recognised by this automaton?

5. Design finite automata that recognise the following languages:
   (a) $\{a^m b^n \mid m = n \mod 3\}$;
   (b) $\{w \in \{a,b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s, modulo } 3\}$.

   (The notation "$m = n \mod 3$" means that the numbers m and n yield the same remainder when divided by three.)

6. Design a finite automaton that recognises sequences of integers separated by plus and minus signs (e.g. 11+20-9, -5+8). Implement your automaton as a computer program that also calculates the numerical value of the input expression.