T-79.148 **Spring 2004**
Introduction to Theoretical Computer Science
Tutorial 13
Problems

Homework problems:

In the first two exercises of this week the aim is to classify the language:

$$L = \{a^i b^j a^j b^i \mid i, j \geq 0\} \ .$$

The hierarchy of formal languages contains the following classes:

(a) regular;
(b) context-free;
(c) context-sensitive;
(d) recursively numberable; and
(e) non-recursively numberable languages.

Let $\mathbf{x}$ be the least class where L belongs.

1. Prove that L belongs to $\mathbf{x}$.
2. Prove that L doesn't belong to any preceeding class.
3. Design a Turing machine that recognises the language $\{a^{2k} \mid k \geq 0\}$, and based on that design an unrestricted grammar whose derivations simulate the computations of your Turing machine. (For the general construction see e.g. the textbook by Lewis & Papadimitriou (2nd Ed. 1998), pp. 230–231, the textbook by Hopcroft & Ullman (1979), pp. 222-223, or search the Internet using keywords "from Turing machines to grammars".) Give a derivation for the sentence aa in your grammar, and

Demonstration problems:

4. Show that all context-sensitive languages can be recognised by linear-bounded automata. (Make use of the fact that in applying the grammar's production rules, the length of the sentential form under consideration can never decrease, except in the special case of the empty string.) Deduce from this result the fact that all context-sensitive languages are recursive.
5. Show that every language generated by an unrestricted grammar can also be generated by a grammar where no terminal symbols occur on the left hand side of any production.
6. Show that every context-sensitive grammar can be put in a normal form where the productions are of the form $S \to \varepsilon$ or $\alpha A \beta \to \alpha \omega \beta$, where A is a nonterminal symbol and $\omega \neq \varepsilon$. (S denotes here the start symbol of the grammar.)