T-79.148 **Spring 2005**
Introduction to Theoretical Computer Science
Tutorial 12, 19-22 April
Problems

Homework problems:

1. *Rice's Theorem.*

   A *semantic property* of a Turing machine is any collection of languages $\mathcal{S}$ over the alphabet $\{0, 1\}$. A Turing machine M has the property $\mathcal{S}$ if $L(M) \in \mathcal{S}$. A property $\mathcal{S}$ is called *trivial* if $\mathcal{S} = \emptyset$ (no Turing machine has the property) or if $\mathcal{S} = RE$ (all Turing machines have the property).

   Show that the property $\mathcal{S} = \{L(M) \mid L(M) \text{ is a regular language}\}$ is non-trivial. Conclude that it is undecidable whether a Turing machine accepts a regular language.

2. Consider application programs running under some given operating system. Let us say that a program P is *dangerous*, if it on some input modifies the operating system's system files. A *general purpose virus tester* is a program that receives as input an arbitrary application program text P, analyses it and returns output "DANGER", if the program is dangerous, and "OK" otherwise. Show that if any dangerous programs exist at all, then general-purpose virus testing is impossible.

3. Design unrestricted grammars (general rewriting systems) that generate the following languages:

   (a) $\{w \in \{a, b, c\}^* \mid w \text{ contains equally many } a\text{'s, } b\text{'s and } c\text{'s}\}$,
   (b) $\{a^{2^n} \mid n \geq 0\}$.

Demonstration problems:

4. Prove, without appealing to Rice's theorem, that the following problem is undecidable:

   Given a Turing machine M; does M accept the empty string?

5. Prove the following connections between recursive functions and languages:

   (i) A language $A \subseteq \Sigma^*$ is recursive ("Turing-decidable"), if and only its characteristic function

   $$\chi_A : \Sigma^* \to \{0, 1\}, \qquad \chi_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{if } x \notin A \end{cases}$$

   is a recursive ("Turing-computable") function.

   (ii) A language $A \subseteq \Sigma^*$ is recursively enumerable ("semidecidable", "Turing-recognisable"), if and only if either $A = \emptyset$ or there exists a recursive function $g : \{0, 1\}^* \to \Sigma^*$ such that

   $$A = \{g(x) \mid x \in \{0, 1\}^*\}.$$