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T-79.148 Introduction to Theoretical Computer Science (2 cr)
Exam Thu 27 Oct 2005, 9–12 a.m.

Write down on each answer sheet:
- Your name, department, and study book number
- The text: "T-79.148 Introduction to Theoretical Computer Science 27.10.2005"
- The total number of answer sheets you are submitting for grading

1. Describe the following languages **both** in terms of regular expressions **and** in terms of deterministic finite automata:
   (a) $\{w \in \{0,1\}^* \mid w$ contains either 010 or 110 (or both) as a substring$\}$, *5p.*
   (b) $\{w \in \{0,1\}^* \mid w$ contains neither 010 nor 110 as a substring$\}$. *5p.*

2. (a) Describe verbally the language generated by the following context-free grammar:
$$\begin{aligned} S &\rightarrow ASb \mid \varepsilon \\ A &\rightarrow aA \mid a \end{aligned}$$
5p.
   (b) Show that the grammar in part (a) is ambiguous. *5p.*
   (c) Design an unambiguous context-free grammar that generates the same language as the grammar in part (a). *5p.*

3. (a) Design a context-free grammar that generates the language
$$L = \{a^m b^n \mid n \geq 0 \text{ and } m = n \text{ or } m = 2n\}.$$
7p.
   (b) Prove (precisely!) that the language in part (a) cannot be described by a regular expression. *8p.*

4. (a) Define the notions of a recursive ("decidable") and recursively enumerable ("semidecidable", "Turing-recognisable") language. What is the main difference between the two notions? *5p.*
   (b) Give an example of a language that is recursively enumerable, but not recursive. (You should provide a precise definition for the language, but need not prove any of its claimed properties.) *5p.*
   (c) Show that if a language $L \subseteq \Sigma^*$ is recursive, then so is its complement language $\overline{L} = \Sigma^* - L$. *5p.*
   (d) Show that if a language $L \subseteq \Sigma^*$ is recursively enumerable but not recursive, then its complement language $\overline{L} = \Sigma^* - L$ is not recursively enumerable. (In proving this claim you may assume as known any auxiliary results that have been presented in your textbook.) *5p.*

Total 60p.