Summary of Network Lifetime and Power Assignment in ad hoc Wireless Networks

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Abstract

In ad-hoc wireless networks, certain network connectivity constraints are of interest because of their practical importance. An example of such a constraint would be *strong connectivity*. The aim is usually to minimize the power used to maintain such connectivities by adjusting the transmission power of the nodes of the network. Such problems are called *Power Assignment* problems. Another set of similar problem classes called *Network Lifetime* problems arise if the nodes have initial battery supply depending on the node and the aim is to maintain a connectivity constraint as long as possible in the network.

Calinescu et al.[1] give approximation algorithms for the *Min-Power Symmetric Connectivity*, *Min-Power Strong Connectivity* and *Min-Power Broadcast* and give a special treatment for the important case of Min-Power Symmetric Connectivity in the Euclidean with node-dependent transmission efficiency. For Network Lifetime, an approximation algorithm is given based on the polynomial time approximation scheme (PTAS) for linear programs by Garg and Köneman.

1 Introduction

Energy efficiency is a central topic on the routing of ad-hoc networks. When discussing wireless networks, it is obivious that the situation becomes much more flexible than in the case of wired networks. The transmission between two nodes can happen either directly or by using intermediate nodes relaying the packets.

It is assumed that the nodes have an adjustable transmission power and an omnidirectional antenna and their position is static. Power required to create a connection between nodes depends on the geometry in which the nodes lie and a node-dependent transmission efficiency. The nodes can adjust their transmission power in order for the network to adhere to a given connectivity constraint and to maximize a nodes lifetime.

The problem consists of a weighted graph $G = (V, E, c)$, where $c : \quad E \rightarrow \mathbb{R}^+$ is the *power requirement function* defined on the set of edges E. A *power* *assignment function* $p: \quad V \to \mathbb{R}^+$ is a function from the vertices V. An edge (u, v) is *supported* by the power assignment p if $p(u) \geq c(u, v)$. The supported subgraph H of G called the *transmission graph* consists of the supported edges of the graph G. The following network connectivity constraints Q of H are considered.

(1) *Strong connectivity* when H is strongly connected

(2) *symmetric connectivity* when the undirected graph having an edge (u, v) iff H has both edges (u, v) and (v, u) must be connected

(3) *broadcast* (resp. *multicast*) from a root $r \in V$, when H contains a directed spanning tree rooted at r (resp. directed Steiner tree for given subset of nodes rooted at r).

1.1 Problem Definitions

The paper discusses two problem classes, the *Power Assignment* problem and the *Network Lifetime* problem. The Power Assignment problem is defined as follows.

Definition 1 *Given a power requrement graph $G = (V, E, c)$ and a connectivity constraint Q, The* Power Assignment *problem is the problem of finding the minimum power assignment function $p: \quad V \to \mathbb{R}^+$ of the minimun total power $\sum_{v \in V} p(v)$ such that the supported subgraph H satisfies the given connectivity constraint Q.*

An equivalent formulation of the Power Assignment problem is the following. Given a directed spanning subgraph H, define the *power* of a vertex u as $p_H(u) = \max_{(u,v) \in E(H)} c(u, v)$ and the *power* of H as $p(H) = \sum_{u \in V} p_H(u)$. The problem then reduces to finding the minimal H satisfying the constraint Q minimizing $p(H)$.

The power assignment problem is a practically interesting problem but it does not take into account the possibly heterogeneous initial battery supply of the nodes or the possibility of dynamically readjusting the power assignment. To answer to the needs of these constraints, the paper introduces the *Network Lifetime* problem. In the problem, an initial battery supply $b(v): \quad V \to \mathbb{R}^+$ is defined on the nodes and the battery function is reduced by amount of $t \cdot p(v)$ for each time period t when the node transmits with power $p(v)$. A *Power Schedule* PT is a set of pairs (p_i, t_i), $i \in \{1, \ldots, m\}$ of power assignment functions p_i and time periods t_i during which the power assignments are used. Power schedule PT is *feasible* if the total amount of energy used by each node v on the whole schedule does not exceed its initial battery supply $b(v)$, that is, $\sum_{i=1}^{m} t_i \cdot p_i(v) \leq b(v) \quad \forall v \in V$. The problem is defined as follows.

Definition 2 *Given a power requirement graph $G = (V, E, c)$, an initial battery supply $b: \quad V \to \mathbb{R}^+$ and a connectivity constraint Q, find a feasible power schedule $PT = \{(p_1, t_1), \ldots, (p_m, t_m)$ of the maximum total time $\sum_{i=1}^{m} t_i$ such that for each power assignment p_i, the supported subgraph H satisfies the given connectivity constraint Q.*

Table 1: Approximation ratios and computational complexities in the asymmetric case

Connectivity constraint	Upper bound	Lower bound
Strong connectivity	$\mathbf{3 + 2 \ln(n - 1)}$	**SCH**
Broadcast	$\mathbf{2 + 2 \ln(n - 1)}$	SCH
Multicast	DST	**DSTH**
Symmetric connectivity	$\mathbf{2 + 2 \ln(n - 1)}$	**SCH**

Table 2: Approximation ratios and computational complexities in the Euclidean with efficiency case

Connectivity constraint	Upper bound	Lower bound
Strong connectivity	$\mathbf{3 + 2 \ln(n - 1)}$	NPH
Broadcast	$\mathbf{2 + 2 \ln(n - 1)}$	NPH
Multicast	DST	NPH
Symmetric Connectivity	**11.73**	NPH

Reformulation of the problem gives the following. Each directed subgraph H satisfying the constraint Q is assigned a real variable $\alpha(H) \leq 0$ and the objective is to maximize $\sum_H \alpha(H)$ while having $\sum_H p_T(u)\alpha(H) \leq b(u) \quad \forall u \in V$.

In addition to the general graph G, three important special cases are studied.

(1) Symmetric case, where $c(u, v) = c(v, u)$

(2) Euclidean case, where $c(u, v) = d(u, v)^\kappa$, where $d(u, v)$ is the Euclidean distance between nodes u and v and κ is the signal attenuation constant, between 2 and 5, and κ is the same for all edges

(3) Single line case, which is a subclass of Euclidean case, where all nodes lie on a single line

The model includes also a *Transmission Efficiency* $e : \quad V \to \mathbb{R}^+$ defined on the nodes $v \in V$. This function constructs a new asymmetric graph G' from the power requirement graph G by redefining the cost c as $c'(u, v) = c(u, v)/e(u)$.

Tables 1, 2 and 3 summarize the results given in the paper for asymmetric, Euclidean with efficiency and symmetric costs respectively. The new results are marked with boldface. Notation SCH means that SET COVER reduces to the problem, DSTH means that DIRECTED STEINER TREE reduces approximation preserving to the problem and DST means that the problem reduces approximation preserving to DIRECTED STEINER TREE. NPH refers to NP-hard and MAXSNPH refers to MAXSNP-hard problems.

In the Upper bound -column the tables give the asymptotic approximation ratio and Lower bound gives the computational complexity of the full problem.

Table 3: Approximation ratios and computational complexitites in the symmetric case

Connectivity constraint	Upper bound	Lower bound
Strong connectivity	2	MAXSNP
Broadcast	$\mathbf{2 + 2}\ln(\mathbf{n - 1})$	SCH
Multicast	$\mathcal{O}(\ln n)$	SCH
Symmetric connectivity	$\frac{5}{3} + \epsilon$	MAXSNPH

2 Algorithms for Asymmetric Power Requirements

The algorithm works by adding structure to the problem greedily. The iteration i starts by a directed subgraph H_i seen as a set of arcs with vertex set V. The strongly connected components of H_i which do not contain the root are the *unhit components* of the graph. An arbitrary node from an unhit component is called a *representative* of the unhit component. The algorithm iterates until no unhit components are found. The structures added to the problem are *spiders* defined below. The best spider is one that gives the biggest reduction in the number of unhit components divided by the weight of the spider. The next graph H_{i+1} is constructed by adding the spider (seen as a set of arcs) to H_i.

Definition 3 *A* Spider *is a directed graph constisting of one vertex called* head *and a set of directed paths (called* feet*) of the spider. The definition allows legs to share vertices and arcs. The* weight *of the spider S, denoted by $w(S)$ is the maximum cost of the arcs leaving the head plus the sum of costs of legs, where the cost of a leg is the sum of the costs of its arcs without the arc leaving the head*

A figure of a spider is given in Figure 1

The weight of a spider S can be higher than $p(S)$ as the legs can share vertices and for those vertices the sum (as opposed to the maximum) of the costs contribute to the weight.

Definition 4 *The* shrink factor*, $sf(S)$ of a spider S with head h is either the number of representatives among its feet if h is reachable (where every vertex is reachable from itself) from the root or if h is not reachable from any of its feet. Otherwise $sf(S)$ is the number of representatives among its feet minus one.*

The algorithm is given in Algorithm 1.

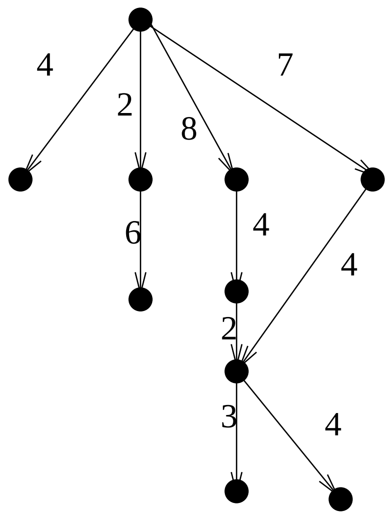

Figure 1: A spider

function *greedy-broadcast*(G, r)
    $H := \emptyset$
    while H has at least one unhit component
        Find the spider S minimizing $w(S)/sf(S)$ w.r.t H
        $H := H \cup S$
    end while
    return H

Algorithm 1: The greedy algorithm for Min-Power Broadcast with asymmetric power requirements

Let $u(H)$ be the amount of unhit component in the graph H. Then the following holds.

Lemma 1 *For a spider S, $u(H_i \cup S) \leq u(H_i) - sf(S)$.*

Fact 1 *Given a spider S, $p(H_i \cup S) \leq p(H_i) + w(S)$.*

A method for finding the spider minimizing the ratio of height and shrink factor is given in [1].

Let OPT be the optimal solution for the Min-Power Broadcast with asymmetric power requirements. Then the following holds.

Lemma 2 *(Existence of a good spider) Given any graph H_i and a set of representatives obtained from H_i, there is a spider S such that $\frac{w(S)}{sf(S)} \leq 2\frac{\mathrm{OPT}}{u(H_i)}$.*

The proof is given in [1]

Theorem 1 *The algorithm in Algorithm 1 has approximation ratio $2 + 2\ln(n-1)$.*

The proof is given in [1]

Table 4: Approximation ratios for $\kappa \in \{2,3,4,5\}$ for the Min-Power Symmetric Connectivity in the Euclidean-with-Efficiency case

κ	r	ratio
2	1.32	11.73
3	1.15	20.99
4	1.08	38.49
5	1.05	72.72

2.1 Min-Power Strong Connectivity with Asymmetric Power Requirements

The case for strong connectivity is similar to the case of broadcast. When v is an arbitrary vertex in a graph, an optimum solution of power OPT consists of an outgoing arborescence A_{out} rooted at v with $p(A_{\text{out}}) \leq \text{OPT}$ and an incoming arborescence A_{in} also $\leq \text{OPT}$.

The broadcast algorithm in previous section produces an outgoing arborescence $B_{\text{out}} \leq (2+2\ln(n-1))A_{\text{out}}$ and Edmonds' algorithm produces a minimum cost arborescence B_{in} rooted at v with the property that $c(B_{\text{in}}) \leq c(A_{\text{in}})$. As $c(A_{\text{in}}) = p(A\text{in}) \leq \text{OPT}$, we have that $p(B_{\text{out}} \cup B_{\text{in}}) \leq p(B_{\text{out}}) + c(B_{\text{in}}) \leq 2(1+\ln(n-1))p(A_{\text{out}}) + c(A_{\text{in}}) \leq (2\ln(n-1)+3)\text{OPT}$. From this we have

Theorem 2 *There is a $2\ln(n-1)+3$-approximation algorithm for strong connectivity with asymmetric power requirements.*

3 Min-Power Symmetric Connectivity in the Euclidean-with-Efficiency Case

In the Euclidean with Efficiency case a cost function $c(u,v)$ is given by the distance $d(u,v)$ of the vertices and the transmission efficiency $e(u)$ of the transmitting node u by

$$c: \quad (u,v) \mapsto \frac{d(u,v)^\kappa}{e(u)}$$

Theorem 3 *Let $w(u,v) = c(u,v) + c(v,u)$. Compute the minimum spanning tree M in this resulting weighted undirected graph. The resulting tree M has*

$$p(M) \leq \min_{r>1} \left(2^\kappa + (r+1)^\kappa + \frac{r^\kappa}{r^\kappa - 1} \right) \text{OPT},$$

where OPT *is the power of the optimum tree.*

The proof is partially presented in [1]. Numerically the approximation ratios are presented in Table 4

4 Network Lifetime

Network Lifetime gets as input a power requrements graph $G = (V, E, c)$ and a battery supply $b : \quad V \to \mathbb{R}^+$. A set $\mathcal{S}$ of directed graphs is given implicitly by the connectivity constraint Q. $|\mathcal{S}|$ is in general exponential in $|V|$. The problem is to maximize

$$\sum_{H \in \mathcal{S}} x_H$$

while maintaining

$$\sum_{H \in \mathcal{S}} p_H(v) x_H \leq b(v), \quad \forall v \in V, x_H \geq 0.$$

Theorem 4 *Even in the special case when all the nodes have the same battery supply, the Network Lifetime for Symmetric Connectivity (or Broadcast or Strong Connectivity) problem is NP-hard in the symmetric power requirements case.*

The proof is outlined in [1].

Using an alteration of the Garg-Köneman -$(1 + \epsilon)$-approximation algorithm, the algorithm for minimum-power assignment can be used to prove the following theorem.

Theorem 5 *For a connectivity constraint and a case of the power requirements graph, given an f-approximation algorithm F for Power Assignment with the given connectivity constraint Q and the case of the power requirements graph with added non-uniform efficiency, there is a $(1 + \epsilon)f$-approximation algorithm for the corresponding Network Lifetime problem.*

The Garg-Köneman Algorithm variation is given in [1].

5 Future work

The authors believe that the following results hold:

(1) Min-Power Steiner Symmetric Connectivity with asymmetric power requirements can be approximated with a $O(logn)$ ratio as suggested in the introduction.

(2) A $(1.35 + \epsilon) \ln n$-algorithm for any $\epsilon > 0$ exists for Min-Power Symmetric Connectivity, Min-Power Stiener Symmetric Connectivity, Min-Power Broadcast and Min-Power Strong Connnectivity.

The existence of efficient exact or constant factor approximation algorithms for Min-Power Broadcast or Min-Power Strong Connectivity in the Euclidean geometry with efficiency is left open. Also the NP-hardness of Network Life in Euclidean case is not known.

An interesting practical problem where there is defined a sensitivity $s(v)$ giving $c(u, v) = \frac{d(u,v)^\kappa}{s(v)}$ could be more studied.

References

[1] G. Calinescu, S. Kapoor, A. Olshevsky, and A. Zelikovsky. Network lifetime and power assignment in ad hoc wireless networks. In *Proc. 11th Ann. European Symp. on Algorithms (ESA 2003)*, pages 114–126, Berlin, 2003. Springer-Verlag.