Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.3001 Logic in Computer Science: Foundations

Examination, May 11, 2007

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

(a) Define the following concepts: *refutation by resolution*, *free variable occurrence*, and *disagreement set*. ($3 \times 2p$)

(b) What is meant by the notation $\Sigma \models \phi$?

Prove in detail that if $\Sigma \cup \{\phi\} \models \psi$, then $\Sigma \models \phi \rightarrow \psi$.

Assignment 2 (10p) Prove the following claims using semantic tableaux:

(a) $\models (A \wedge B) \vee (\neg A \wedge C) \rightarrow \neg(A \wedge \neg B) \wedge (\neg C \rightarrow A)$

(b) $\models \exists x(P(x) \vee Q(x)) \leftrightarrow \exists x P(x) \vee \exists x Q(x)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg(\exists x \forall y G(y,x) \rightarrow \forall y \exists x G(y,x)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us consider the contents of a book shelf which is described using a binary predicate $C(x,y)$ = "books x and y are located consecutively on the shelf". Suppose that b, c and d are three constants referring to specific books authored by Böll, Carr, and Dostojevski, respectively.

(a) Define a ternary predicate $B(x,y,z)$ = "book y appears between books x and z, i.e., after x but before z on the shelf" using predicate logic so that your definition covers all books on an individual shelf.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \cup \{C(b,c), C(c,d)\} \not\models B(d,c,b).$$

Assignment 5 (10p)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if(}B\texttt{) then \{}C_1\texttt{\} else \{}C_2\texttt{\}}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Divide`:

$$\texttt{v=0 ; z=x ; while(z>=y) \{z=z-y ; v=v+1\}}.$$

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Divide}\ [\texttt{v==x/y}],$$

where `x/y` denotes the integer quotient when `x` is divided by `y`.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.3001/`
Please remember the last time tracking questionnaire!