T-79.3001 Logic in Computer Science: Foundations

Examination, January 4, 2008

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10*p*)

(a) Define the following concepts: *disjunctive normal form*, *complete proof system*, and *most general unifier*. ($3 \times 2p$)

(b) What is meant by the notation $\phi \underline{\vee} \psi$?

Prove in detail that if $\models \phi \underline{\vee} \psi$, then $\models \neg\phi \underline{\vee} \neg\psi$.

Assignment 2 (10*p*) Prove the following claims using semantic tableaux:

(a) $\models (A \rightarrow B \vee C) \leftrightarrow (\neg B \wedge \neg C \rightarrow \neg A)$

(b) $\{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)\} \models \exists z Q(z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10*p*) Derive a Prenex normal form and a clausal form (i.e. a set of clauses S) for the sentence

$$\neg \forall x \exists y (\exists z R(y,z) \rightarrow \exists v R(x,v)).$$

Try to make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10*p*) Let us represent strings “”, “*a*”, “*b*”, “*aa*”, “*ab*”, “ba”, “bb”, …that consist of letters *a* ja *b* using ground terms

$$e,\ a(e),\ b(e),\ a(a(e)),\ a(b(e)),\ b(a(e)),\ b(b(e)),\ \ldots,$$

built of a constant symbol e, which represents the empty string “”, and unary functions $a(x)$ and $b(x)$, that append the respective letter a or b at the beginning of a string x. Thus $a(b(e))$ is interpreted as $a(b(\text{“”})) = a(\text{“}b\text{”}) = \text{“}ab\text{”}$.

(a) Define predicate $AB(x) =$ “the string x is of the form $abab \ldots ab$ where the string ab repeats $n \geq 0$ times” using predicate logic so that your definition covers all finite strings represented as explained above.

(b) Give a model $\mathcal{S} \models \Sigma$ of your definition Σ on the basis of which it holds that

$$\Sigma \not\models AB(b(a(e))).$$

Assignment 5 (10*p*)

Explain how the *weakest precondition* B_1 of an if-statement

$$\texttt{if}(B)\ \texttt{then}\ \{C_1\}\ \texttt{else}\ \{C_2\}$$

can be formed given a postcondition B_2 for it.

Consider the following program `Divide`:

```
v=0;z=x;while(z>=y){z=z-y;v=v+1}.
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{Divide}\ [\texttt{v==x/y}],$$

where `x/y` denotes the integer quotient when `x` is divided by `y`.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.