T-79.3001 Logic in computer science: foundations **Spring 2007**
Exercise 2 (propositional logic 2.1 – 3.5)
January 30–February 1, 2007

Tutorial problems

1. Give definitions for the connectives in propositional logic using conjunction ($\wedge$) and negation ($\neg$).

2. a) Let $\mathcal{A} = \{A, C\}$ be a truth assignment. Find the truth value of

$$C \wedge (A \leftrightarrow B) \rightarrow ((A \vee \neg B) \wedge (B \vee \neg A) \rightarrow C)$$

by using (i) truth tables and (ii) the definition of truth values. What can be said about the validity, satisfiability, and unsatisfiability of the proposition?

b) Apply truth tables to see whether $\{A \rightarrow B, B \rightarrow C\} \models A \rightarrow C$ holds.

3. Let $\mathcal{A}$ be a truth assignment. Prove by induction that for all propositional statements ϕ, either $\mathcal{A} \models \phi$ or $\mathcal{A} \not\models \phi$.

Demonstration problems

4. Give definitions for the connectives in propositional logic using

a) the proposition that is always false ($\bot$) and implication ($\rightarrow$), and

b) the Sheffer stroke.

5. List all possible binary connectives (16 in total) and give their definitions using the basic connectives in propositional logic.

6. Let $\mathcal{A}_1 \subseteq \mathcal{P}$ and $\mathcal{A}_2 \subseteq \mathcal{P}$ be truth assignments and $\phi \in \mathcal{L}$ a proposition. Show that if $\mathcal{A}_1 \cap At(\phi) = \mathcal{A}_2 \cap At(\phi)$, then $\mathcal{A}_1 \models \phi \iff \mathcal{A}_2 \models \phi$.

7. Let $\mathcal{A} = \emptyset$ be a truth assignment. Find the truth value of

$$(\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$$

by using

a) the truth table and

b) the definition of truth values.

8. An engineer designed a specification for two traffic light posts positioned in the intersection of two one-way streets:

(i) Both the light posts have a green, a yellow and a red light. Exactly one of the lights in each light post is lit at all times.

(ii) Both green lights are not lit at the same time.

(iii) If one lamp post has the red light on, then the other has either the green or the yellow light on.

a) Formalize the above requirements as a set of propositional statements.

b) Construct a truth table for the set of statements.

c) Give (i) a model for the set of statements, and (ii) a truth assignment such that the set of statements is not satisfied.

d) Are the requirements complete enough for a real life situation?

9. Apply truth tables to see whether the following claims hold.

a) $(A \to B) \to ((B \to C) \to (A \to C))$ is valid.

b) $\neg((A \to B) \to ((\neg A \to B) \to B))$ is unsatisfiable.

c) $A \leftrightarrow B$ and $\neg(A \leftrightarrow \neg B)$ are logically equivalent.

d) $\{(A \wedge B) \vee (C \wedge A), (A \wedge B) \vee \neg B\} \models A \vee (C \wedge \neg B)$.