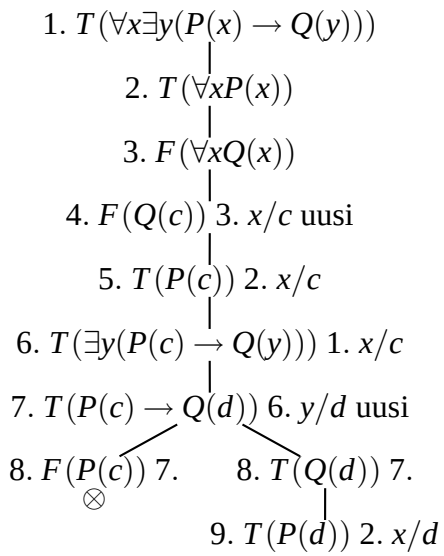
T-79.3001 Logic in computer science: foundations **Spring 2008**
Exercise 9 ([NS, 1997], Predicate Logic, Chapters 6 – 7)
April 9–11, 2008

Solutions to demonstration problems

Solution to Problem 4

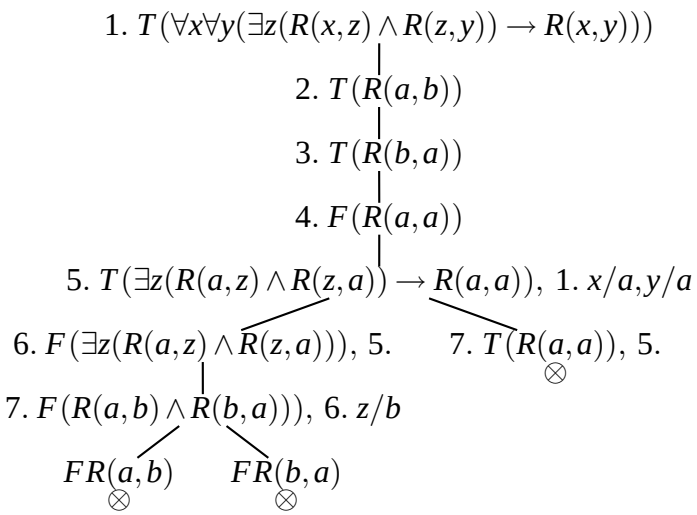
a) Tableau proof:



It seems that the tableau cannot be finished. We read a counter-example $\mathcal{S}$ from an open branch: domain $U = \{1,2\}$, interpretations for constants $c^{\mathcal{S}} = 1$ and $d^{\mathcal{S}} = 2$, and interpretations for predicates $P^{\mathcal{S}} = \{1,2\}$ and $Q^{\mathcal{S}} = \{2\}$.

Since the *tableau is not finished*, we *need to check the counter-example*. Now, we get $\mathcal{S} \models \forall x \exists y(P(x) \to Q(y))$, $\mathcal{S} \models \forall x P(x)$ and $\mathcal{S} \not\models \forall x Q(x)$ for $\mathcal{S}$.

b) Tableau proof:



All the branches in the tableau are contradictory and thus the claim holds.

Solution to Problem 5
We choose the following predicates:

$$\begin{array}{lcl} G(x) & = & \text{“}x\text{ is guilty”,} \\ L(x) & = & \text{“}x\text{ is liar”,} \\ A(x) & = & \text{“}x\text{ is accused”, and} \\ W(x) & = & \text{“}x\text{ is witness”.} \end{array}$$

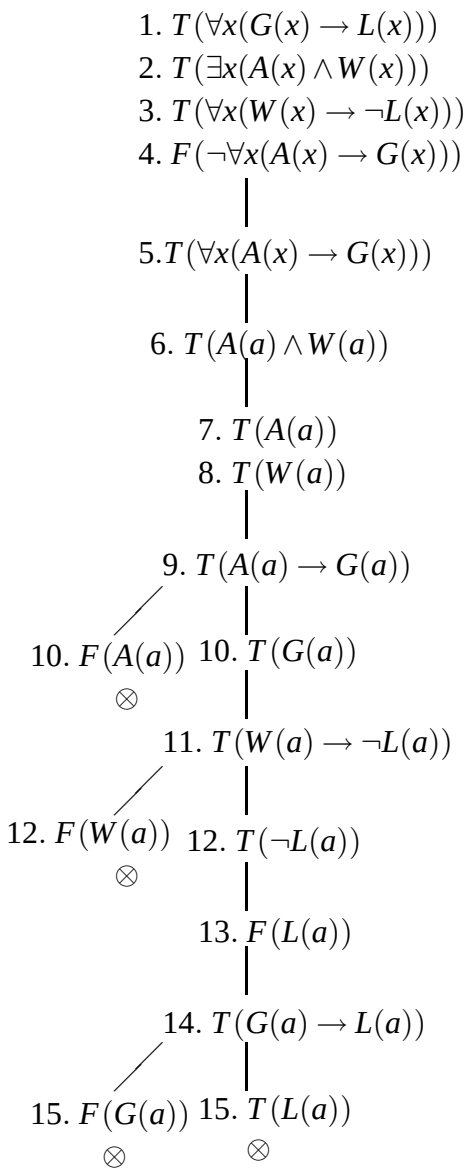
The sentences are:

(i) $\forall x(G(x)\rightarrow L(x))$,

(ii) $\exists x(A(x)\wedge W(x))$, ja

(iii) $\forall x(W(x)\rightarrow \neg L(x))$.

and we want to show that $\neg\forall x(A(x)\rightarrow G(x))$. The tableaux proof is as follows.


1. $T(\forall x(G(x) \rightarrow L(x)))$
2. $T(\exists x(A(x) \wedge W(x)))$
3. $T(\forall x(W(x) \rightarrow \neg L(x)))$
4. $F(\neg\forall x(A(x) \rightarrow G(x)))$
5. $T(\forall x(A(x) \rightarrow G(x)))$
6. $T(A(a) \wedge W(a))$
7. $T(A(a))$
8. $T(W(a))$
9. $T(A(a) \rightarrow G(a))$
10. $F(A(a))$
$\otimes$
10. $T(G(a))$
11. $T(W(a) \rightarrow \neg L(a))$
12. $F(W(a))$
$\otimes$
12. $T(\neg L(a))$
13. $F(L(a))$
14. $T(G(a) \rightarrow L(a))$
15. $F(G(a))$
$\otimes$
15. $T(L(a))$
$\otimes$

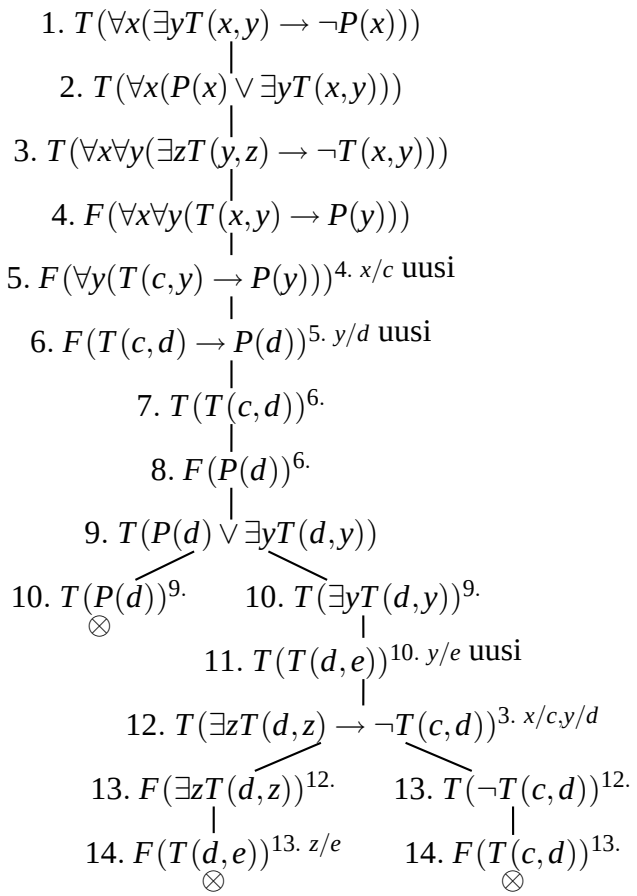
Solution to Problem 6
We use the following predicates:

$$\begin{array}{lcl} T(x,y) & = & \text{“brick } x \text{ is on brick } y\text{”, and} \\ P(x) & = & \text{“brick } x \text{ is on the table”.} \end{array}$$

The set of sentences is:

$$\{\forall x\,(\exists y\,T(x,y) \rightarrow \neg P(x)),\ \forall x\,(P(x) \vee \exists y\,T(x,y)),\\ \forall x \forall y\,(\exists z\,T(y,z) \rightarrow \neg T(x,y))\}$$

and we want to show that $\forall x \forall y\,(T(x,y) \rightarrow P(y))$.
Tableau proof:



Note: 1) can be equivalently stated as $\forall x \forall y(T(x,y) \rightarrow \neg P(x))$ and 3) as $\forall x \forall y \forall z(T(y,z) \rightarrow \neg T(x,y))$. How would the tableau look if you used these sentences?