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T-79.4201 Search Problems and Algorithms (4 cr)
Exam 3 Mar 2007, 10 a.m.–1 p.m.

Write down on each answer sheet:
- Your name, department, and student number
- The text: "T-79.4201 Search Problems and Algorithms 3.3.2007"
- The total number of answer sheets you are submitting for grading

1. Consider the following NP-complete VERTEX COVER problem:

   INSTANCE: Undirected graph $G = (V,E)$.
   QUESTION: Find a minimal size subset of nodes (vertices) of G, $C \subseteq V$, such that C covers at least one end of each edge of G, i.e.. if $\{u,v\} \in E$, then either $u \in C$ or $v \in C$ or both.

   Present in pseudocode form a simulated annealing approach to finding good vertex covers for a given graph. Describe particularly clearly: (a) what are the candidate solutions considered by your method and what is their neighbourhood relation, (b) what is the cost (objective) function associated to the candidate solutions, (c) how does one choose the next solution for consideration from the neighbourhood of a given candidate solution, and (d) how does one choose the initial candidate solution for the computation. The details of the cooling schedule are not important for the purposes of this problem, as long as the principle is presented correctly. (E.g. some simple geometric, i.e. constant factor, cooling scheme is quite appropriate.)

2. a) Give a Boolean circuit that computes the Boolean function $f(x_1,x_2,x_3)$ in the table on the right.
   b) Describe how the Min Conflict Heuristic (MCH) algorithm for solving constraint satisfaction problems works and simulate the algorithm for three local step when the input is the constraint satisfaction problem

   $$\langle C(x,y), C(y,z), C_1(z,x); x \in \{1,2,3\}, y \in \{1,2,3\}, z \in \{1,2,3\}\rangle$$

   where $C = \{(1,2),(1,3),(2,3)\}$.

x_1	x_2	x_3	f
0	0	0	1
0	1	0	1
1	0	0	0
1	1	0	1
0	0	1	1
0	1	1	0
1	0	1	1
1	1	1	1

3. a) Express the condition "$x+5 \geq 0$ or $x+15 \leq 0$" as a set of linear constraints when $-1000 \leq x \leq 1000$.
   b) Consider the following linear programming problem:

   $$\begin{aligned}&\max\ x_1 + 2x_2 \text{ s.t.}\\ &x_1 - 5x_2 \geq 7\\ &-x_1 + x_2 \leq -3\\ &x_1 \geq 0\end{aligned}$$

   Transform the problem into the standard form and give a basic feasible solution for the problem in the standard form.

4. Describe how the A* algorithm could be used for the exact solution of the VERTEX COVER problem discussed in Problem 1, i.e. for the systematic search for a minimum size vertex cover in a given graph. You do not need to repeat the details of the A* algorithm itself, but the following must be presented carefully: (a) how are the nodes and edges in the search graph employed by the A* algorithm determined, (b) what is the start node of the search and what are the goal nodes, and (c) how are the edge costs in the search graph determined. Present also some nontrivial admissible distance heuristic $h(x)$ that could conceivably be used to direct the search. ("Nontrivial" means here that the heuristic must differ in some interesting way from the trivial estimate $h(x) \equiv 0$.)

Grading: Each problem 10p, total 40p.

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