T–79.4301 Parallel and Distributed Systems (4 ECTS)

T–79.4301 Rinnakkaiset ja hajautetut järjestelmät (4 op)

Lecture 10

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Other models of Concurrency

- Process algebras - An algebraic way of compactly specifying LTSs. Example specifying two synchronizing LTSs: $I = ((a.(\tau.c.0 + b.0)) \parallel (a.b.0))$, where "||" is parallel composition, "." is sequential composition, "+" in non-deterministic choice, and "0" is a deadlocking process. Lots of variants exist, the most well know are CCS and CSP.
- Petri nets - A model of concurrency developed by C.A. Petri in 1962. Also lots of variants exist.
- Extended finite state machines, SMV programs (input language of the NuSMV model checker), . . .

Petri nets

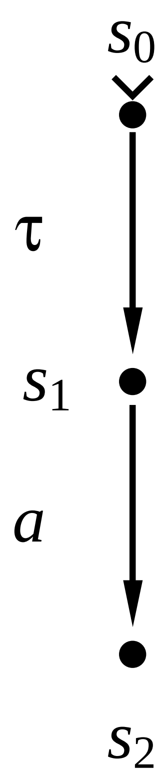
For another perspective into models of concurrency, consider Petri nets. The class we use are called place/transition nets (P/T-nets). A P/T-net is a tuple $N = (P, T, F, W, M_0)$, where

- P is a finite set of places,
- T is a finite set of transitions,
- $F \subseteq (P \times T) \cup (T \times P)$ is the flow relation,
- $W : F \mapsto \mathbb{N} \setminus \{0\}$ is the arc weight mapping, and
- $M_0 : P \mapsto \mathbb{N}$ is the initial marking.

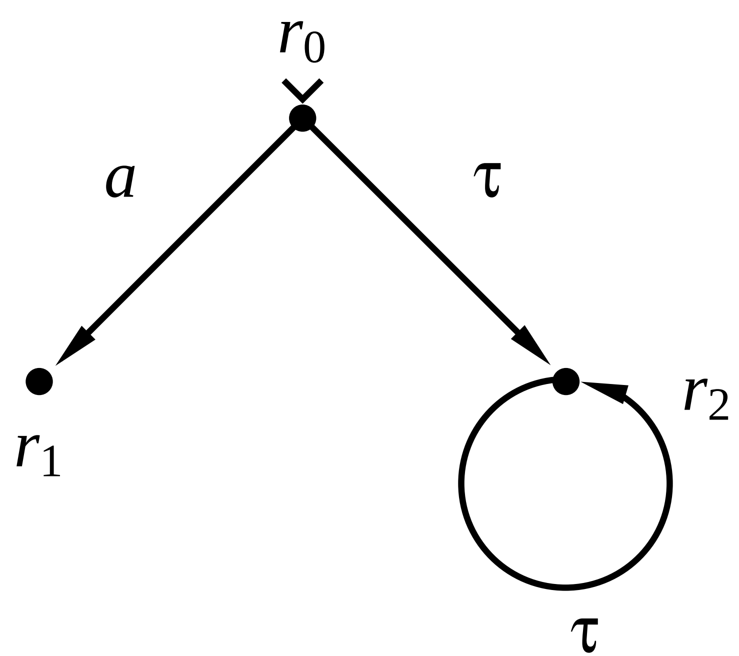
Running Example

Recall the synchronization of LTSs from Lecture 6:

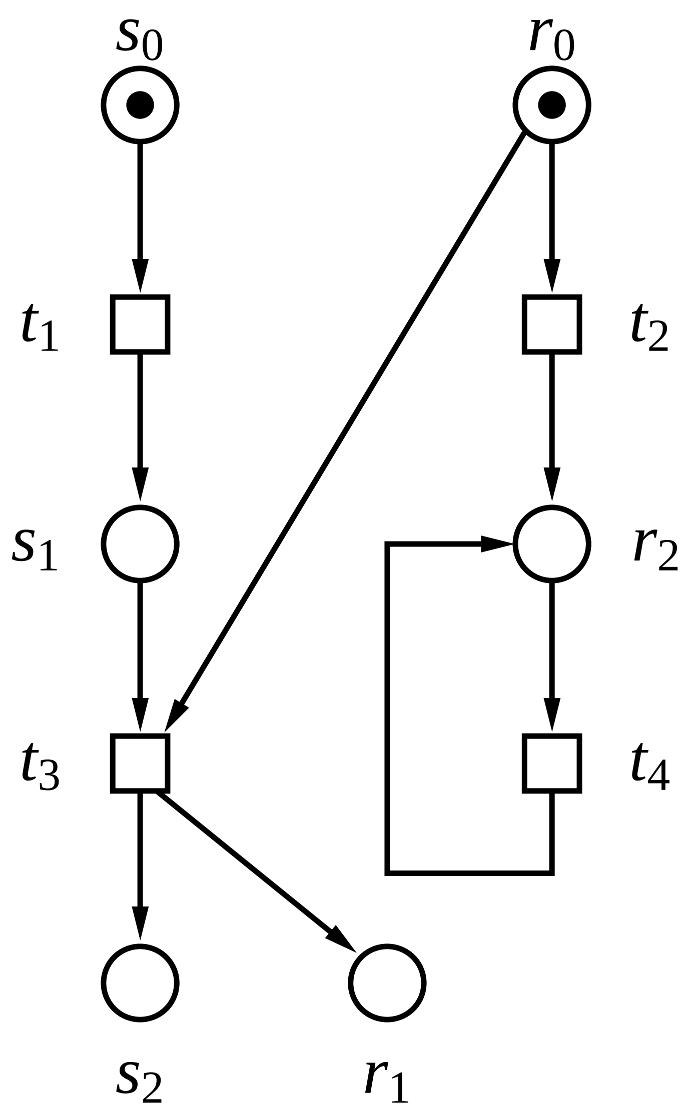
$L_1: \quad \Sigma_1 = \{a\}$



$L_2: \quad \Sigma_2 = \{a\}$



Running Example as P/T net



The running Example

- Places $P = \{s_0, s_1, s_2, r_0, r_1, r_2\}$.
- Transitions $T = \{t_1, t_2, t_3, t_4\}$.
- Flow relation $F = \{(s_0, t_1), (t_1, s_1), (r_0, t_2), (t_2, r_2), (s_1, t_3), (r_0, t_3), (t_3, s_2), (t_3, r_1), (r_2, t_4), (t_4, r_2)\}$.
- Arc weight mapping $W(x, y) = 1$ for all $(x, y) \in F$. We use the convention that only arcs weights $W(x, y) > 1$ are drawn next to the arc (x, y), i.e., the default arc weight is 1.
- Initial marking $M_0 = \{s_0 \mapsto 1, s_1 \mapsto 0, s_2 \mapsto 0, r_0 \mapsto 1, r_1 \mapsto 0, r_2 \mapsto 0\}$.

From LTSs to P/T-nets

Intuition behind the mapping:

- Local states of the components are mapped to places.
- Transitions of the Petri net consist of all legal ways of synchronizing the local transitions of the components. (Potential size blow-up here!)
- The flow relation records what is the precondition under which the synchronization can happen, and what is the effect of the synchronization on the state of each component.
- The initial marking records the initial state of the components.

From LTSs to P/T-nets

- Given $L = L_1 || L_2 || \cdots || L_n$ with $L_i = (\Sigma_i, S_i, S_i^0, \Delta_i)$, we get a P/T-net N_L as follows:
- $P = S_1 \cup S_2 \cup \cdots \cup S_n$,
- $T \subseteq \Delta_1 \cup \{-\} \times \Delta_2 \cup \{-\} \times \cdots \times \Delta_n \cup \{-\}$ (to be defined on the next slide),
- F is the smallest relation satisfying for every (P/T-net) transition $g \in T$:
  - For all $1 \leq i \leq n, t_j = (p, l, p') \in \Delta_i$: If $g = (\ldots, t_j, \ldots)$ then $(p, g) \in F$ and $(g, p') \in F$.
- $M_0(p) = 1$ if $p \in S_1^0 \cup S_2^0 \cup \cdots \cup S_n^0$, and $M_0(p) = 0$ otherwise.

From LTSs to P/T-nets (cnt.)

- For all $x \in \Sigma \cup \{\tau\}$ and all $g \in \Delta_1 \cup \{-\} \times \Delta_2 \cup \{-\} \times \cdots \times \Delta_n \cup \{-\}$ the (P/T-net) transition $g = (t_1, t_2, \ldots, t_n) \in T$ iff:
  - $x = \tau$: there is $1 \leq i \leq n$ such that $t_i = (s_i, \tau, s_i') \in \Delta_i$ and $t_j = -$ for all $1 \leq j \leq n$, when $j \neq i$.
  - $x \neq \tau$: for every $1 \leq i \leq n$: $t_i = (s_i, x, s_i') \in \Delta_i$, when $x \in \Sigma_i$ and $t_i = -$, when $x \notin \Sigma_i$.

Finally we define $W(x, y) = 1$ for all $(x, y) \in F$.

From LTSs to P/T-nets (cnt.)

- We now claim that reachability graphs of $L = L_1 || L_2 || \cdots || L_n$ and N_L are the same.
- However, to do so we have to define the behavior of P/T-nets.

Behavior of P/T-nets

- The state of a P/T-net consist of a *marking* $M : P \mapsto \mathbb{N}$, which tells for each place how many *tokens* (drawn as black dots) it contains.
- The notation $M(p)$ denotes the number of tokens in place p.
- In our running example $M(p) \leq 1$ for all places $p \in P$, i.e., each place contains at most one token. However, this is not required in general.

Behavior of P/T-nets

- The *preset* of a node $x \in P \cup T$ is denoted by ${}^\bullet x$ and defined to be: ${}^\bullet x = \{y \in P \cup T \mid (y, x) \in F\}$.
  The preset of a node consist of those nodes from which an arc to x exist. In our running example ${}^\bullet t_3 = \{s_1, r_0\}$.
- The *postset* of a node $x \in P \cup T$ is denoted by $x^\bullet$ and defined to be: $x^\bullet = \{y \in P \cup T \mid (x, y) \in F\}$.
  The postset of a node consist of those nodes to which an arc from x exist. In our running example $t_3{}^\bullet = \{s_2, r_1\}$.

Enabling of transitions

- To simplify definitions, we extend $W(x,y)$ to all pairs $(x,y) \in (P \cup T) \times (T \cup P)$ as follows: if $(x,y) \notin F$ then $W(x,y) = 0$.
- A transition $t \in T$ is enabled in marking M, denoted $t \in enabled(M)$, iff for all $p \in P : M(p) \geq W(p,t)$. (All places p which are in the preset of t contain at least the number of tokens specified by $W(p,t)$.)

Firing of transitions

- The marking M' reached after firing t, denoted $M' = \mathit{fire}(M,t)$, is defined for all $p \in P$ as:
  $M'(p) = M(p) - W(p,t) + W(t,p)$.
  (First remove as many tokens as given by $W(p,t)$ from all places in the preset of t, and then add as many tokens for all places in the postset of t as denoted by $W(t,p)$.)

Reachability graph

Analogous to the similar definition for LTSs (from end of Lecture 5): Reachability graph $G = (V, E, M_0)$ is the graph with the smallest sets of nodes V and edges E such that:

- $M_0 \in V$, where M_0 is the initial marking of the net N, and
- if $M \in V$ then for all $t \in enabled(M)$ it holds that $M' = fire(M, t) \in V$ and $(M, t, M') \in E$.

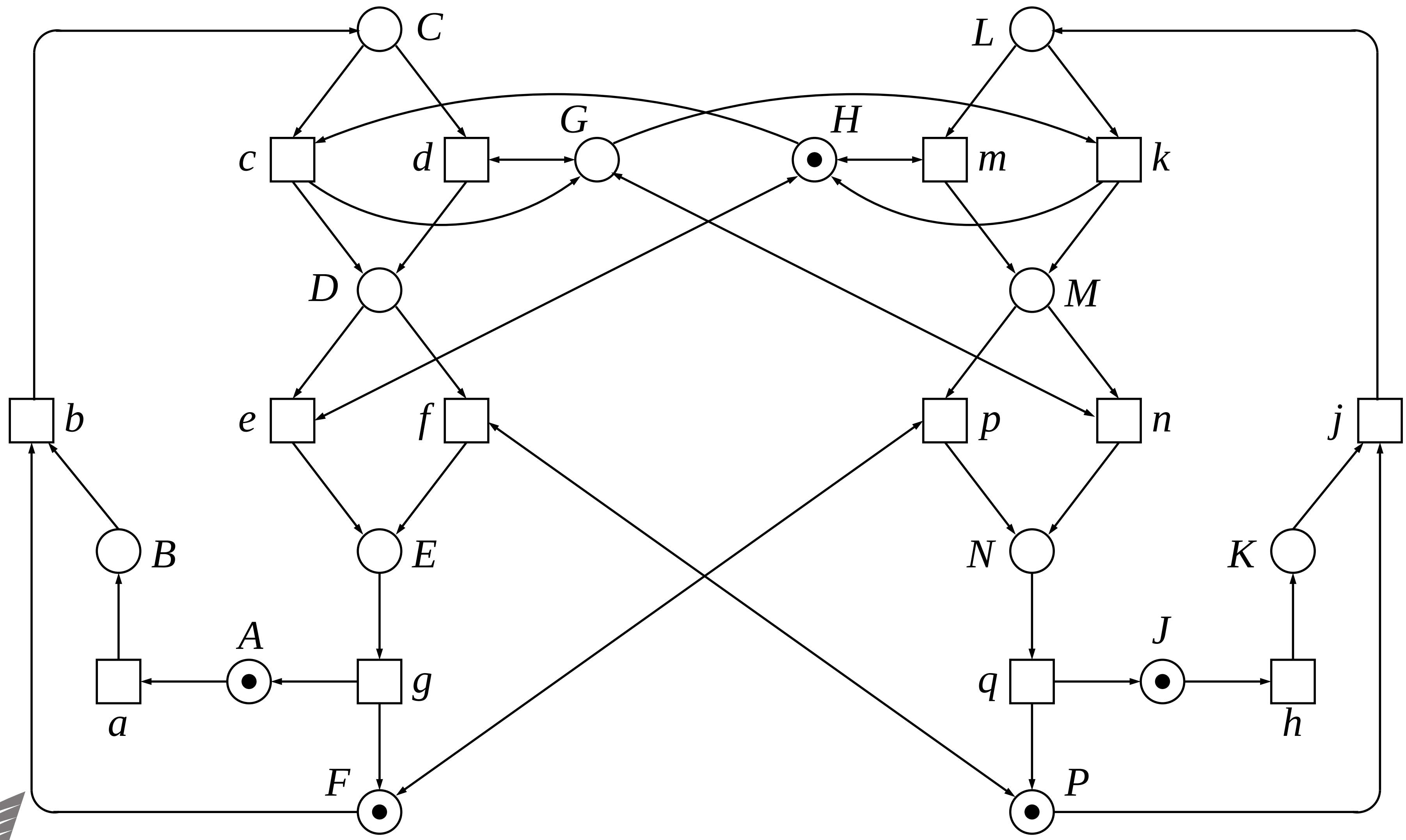
Reachability graph (cnt.)

- It is easy to define a P/T-net with an infinite reachability graph.
- A place $p \in P$ is defined to be k-bounded iff for all reachable markings $M \in V$ it holds that $M(p) \leq k$.
- A net is defined to be k-bounded if all its places are k-bounded
- A net is defined to be *unbounded* (i.e., infinite state) iff it is not k-bounded for any $k \in \mathbb{N}$.

P/T-nets and Turing machines

- It is *not* possible to simulate a Turing machine with a P/T-net. Asking whether a marking M is reachable is in fact decidable for P/T-nets (even with infinite reachability graphs).
- The algorithms used are quite involved, and we do not know of an implementation of the theoretical result in question.
- There is a simple (but slow in the worst case) algorithm which can compute which places of the net are unbounded, called the coverability graph algorithm.

Peterson's Mutex (by W. Reisig)



From 1-bounded P/T-nets to LTSs

- A 1-bounded P/T-net N with $|P|$ places can always be converted to a synchronization of LTSs $L_N = L_1 || L_2 || \cdots || L_n$ with $n \leq |P|$ components which have two states each. The reachability graph of L_N will be isomorphic to that of N.
- The construction is slightly too complcated to show here. The main trick is to use the set of transitions T as the alphabet Σ in L_N, and to make each L_i corresponding to a place $p \in P$ synchronize on all labels $t \in {}^\bullet p \cup p^\bullet$.

From P/T-nets to Promela

- Suppose that the net N we are looking is 255-bounded. Holzmann suggests the following scheme for translating P/T-nets (with $W(x,y) = 1$ for all $(x,y) \in F$, a restriction which can be easily removed) to Promela as shown in the next two slides.

From P/T-nets to Promela (cnt.)

```
#define Place byte /* < 256 tokens per place */

Place s0, s1, s2, r0, r1, r2;

#define inp1(x)          (x>0) -> x--
#define inp2(x,y)        (x>0&&y>0) -> x--; y--

#define out1(x)          x++
#define out2(x,y)        x++; y++
```

From P/T-nets to Promela (cnt.)

```
init
{
            atomic {s0=1;r0=1} /*initial marking*/
            do
/* t1 */    :: atomic { inp1(s0)   -> out1(s1) }
/* t2 */    :: atomic { inp1(r0)   -> out1(r2) }
/* t3 */    :: atomic { inp2(s1,r0)-> out2(s2,r1)}
/* t4 */    :: atomic { inp1(r2)   -> out1(r2) }
            od
}
```

From P/T-nets to Promela (cnt.)

- Actually, all `atomic` statements of the translation can safely be replaced with `d_step` statements.
- By using the LTS to P/T-net mapping first also LTSs can be translated to Promela.

From P/T-nets to Promela (cnt.)

- It may be more efficient to use a Petri net model checker such as PROD (`http://www.tcs.hut.fi/Software/prod/`) to do the model checking as for example the partial order reductions in Spin are not really effective for the model obtained from the translation. (The concurrency of the model is hidden inside the data manipulation of a single process.)
- Another Petri net model checker is Maria (`http://www.tcs.hut.fi/Software/maria/index.en.html`).
- Both of the tools actually use high-level Petri nets, which contain extensions to deal with structured data

Structural Analysis via Example

We want to prove mutual exclusion of Peterson's mutex algorithm. The critical sections correspond to places E and N, and thus our proof objective is:

$$M(E) + M(N) \leq 1 \tag{1}$$

We can easily check that the net satisfies the following place invariants as they hold in the initial state and are preserved by every transition:

$$M(C) + M(D) + M(E) + M(F) = 1 \tag{2}$$
$$M(G) + M(H) = 1 \tag{3}$$
$$M(L) + M(M) + M(N) + M(P) = 1 \tag{4}$$

Example (cnt.)

By linear algebra, we can sum up the invariants (2), (3), and (4) to obtain a new invariant:

$$M(C) + M(D) + M(E) + M(F) + M(G) + M(H) + M(L) + M(M) + M(N) + M(P) = 3 \quad (5)$$

We need expressions on the markings which do not use equality to a constant on the right hand side to proceed further.

Example (cnt.)

It is easy to check that the following equation holds in the initial state and is preserved by every transition:

$$M(C) + M(F) + M(G) + M(M) \geq 1 \qquad (6)$$

Next subtract (6) from (5), to get the result:

$$M(D) + M(E) + M(H) + M(L) + M(N) + M(P) \leq 2 \qquad (7)$$

Example (cnt.)

We also have:

$$M(D) + M(H) + M(L) + M(P) \geq 1 \tag{8}$$

When we subtract (7) from (6), we get the result:

$$M(E) + M(N) \leq 1 \tag{9}$$

Now, (8) is our proof objective (1), and thus we are done. Therefore the mutual exclusion property holds for the Peterson's mutex algorithm.