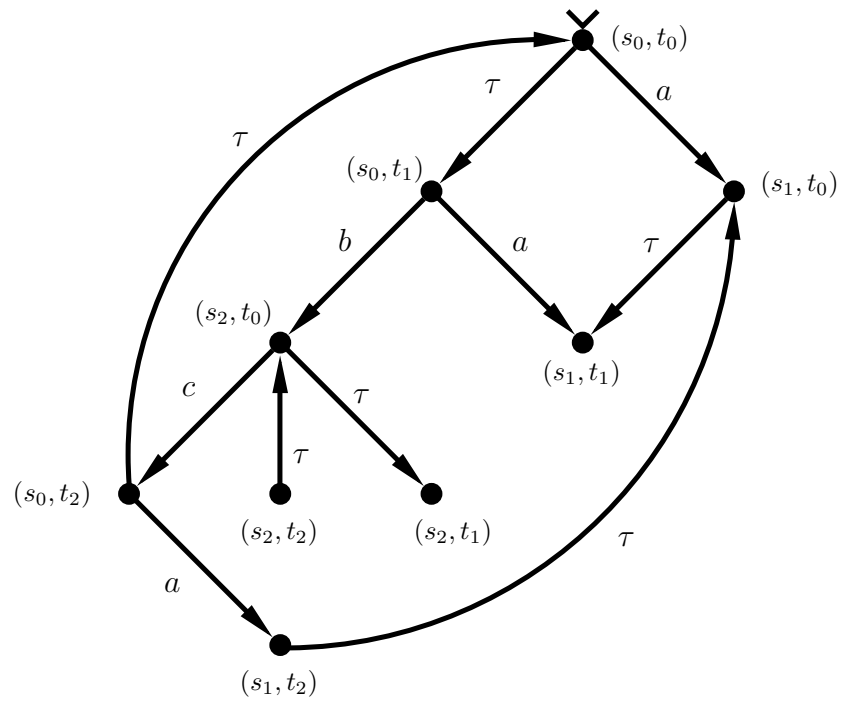1. a) $L_1 \| L_2 = (\Sigma_{12}, S_{12}, S_{12}^0, \Delta_{12})$, where

$$\Sigma_{12} = \Sigma_1 \cup \Sigma_2 = \{a, b, c\},$$
$$S_{12} = S_1 \times S_2 = \big\{(s_0, t_0), (s_0, t_1), \ldots, (s_2, t_2)\big\},$$
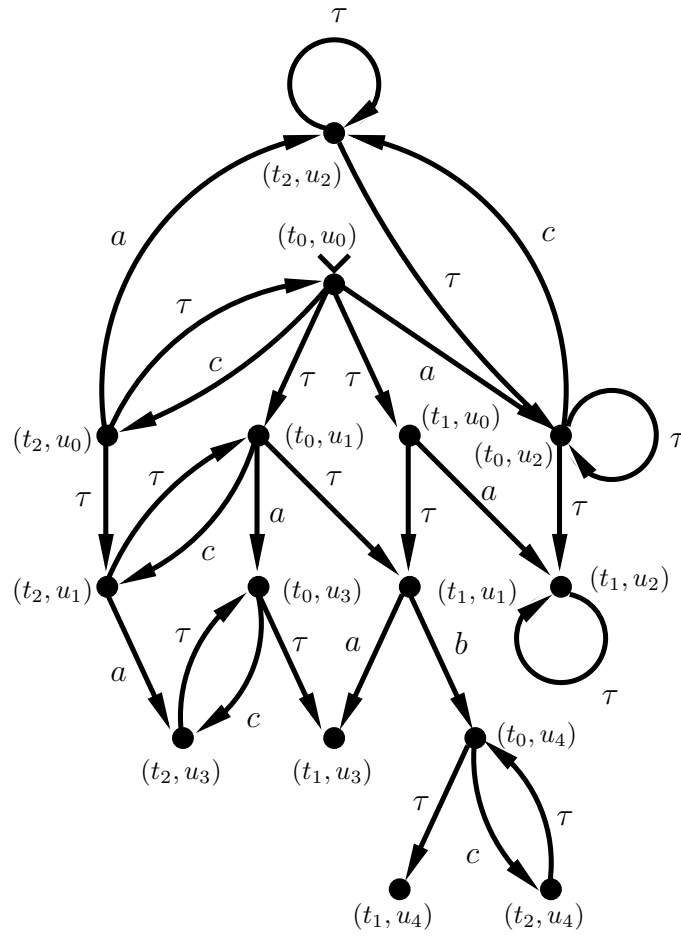$$S_{12}^0 = S_1^0 \times S_2^0 = \big\{(s_0, t_0)\big\},$$

and the transition relation Δ_{12} is defined as shown in the figure below.

b) $L_2 \,\|\, L_3 = (\Sigma_{23}, S_{23}, S_{23}^0, \Delta_{23})$, where

$$\Sigma_{23} = \Sigma_2 \cup \Sigma_3 = \{a, b, c\},$$
$$S_{23} = S_2 \times S_3 = \big\{(t_0, u_0), (t_0, u_1), \ldots, (t_2, u_4)\big\},$$
$$S_{23}^0 = S_2^0 \times S_3^0 = \big\{(t_0, u_0)\big\}, \text{ and}$$

Δ_{23}:

τ

(t_2, u_2)

a (t_0, u_0) c

τ τ

c τ τ a

(t_2, u_0) (t_0, u_1) (t_1, u_0)

(t_0, u_2) τ

τ τ τ a a τ

c

(t_2, u_1) (t_0, u_3) (t_1, u_1) (t_1, u_2)

τ a τ a b τ

(t_2, u_3) (t_1, u_3)

c

(t_0, u_4)

τ τ

c

(t_1, u_4) (t_2, u_4)

c) $L_1 \parallel L_2 \parallel L_3 = (\Sigma_{123}, S_{123}, S_{123}^0, \Delta_{123})$, where

$$\Sigma_{123} = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 = \{a, b, c\},$$
$$S_{123} = S_1 \times S_2 \times S_3 = \{(s_0, t_0, u_0), (s_0, t_0, u_1), \ldots, (s_2, t_2, u_4)\},$$
$$S_{123}^0 = S_1^0 \times S_2^0 \times S_3^0 = \{(s_0, t_0, u_0)\}, \text{ and}$$

Δ_{123}: