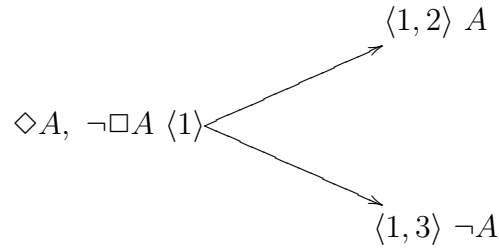




2. a)

1.  $\langle 1 \rangle \neg(\Diamond A \rightarrow \Box A)$
2.  $\langle 1 \rangle \Diamond A$  (1)
3.  $\langle 1 \rangle \neg \Box A$  (1)
4.  $\langle 1, 2 \rangle \neg \neg A$  (2;  $\Diamond$  on lyhennysmerkkintä  $\neg \Box \neg$ :lle)
5.  $\langle 1, 2 \rangle A$  (4)
6.  $\langle 1, 3 \rangle \neg A$  (3)

Ei **K**-pätevä.



b)

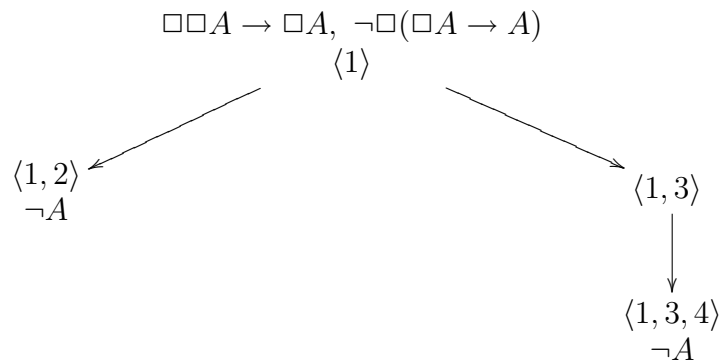
1.  $\langle 1 \rangle \neg(\Diamond \Box A \vee \Box \Diamond \neg A)$
  2.  $\langle 1 \rangle \neg \Diamond \Box A$  (1)
  3.  $\langle 1 \rangle \neg \Box \Diamond \neg A$  (1)
  4.  $\langle 1 \rangle \Box \neg \Box A$  (2)
  5.  $\langle 1, 2 \rangle \neg \Diamond \neg A$  (3)
  6.  $\langle 1, 2 \rangle \neg \Box A$  (4)
  7.  $\langle 1, 2 \rangle \Box \neg \neg A$  (5)
  8.  $\langle 1, 2, 3 \rangle \neg A$  (6)
  9.  $\langle 1, 2, 3 \rangle \neg \neg A$  (7)
- ⊗

Lause on **K**-pätevä.

c)

1.  $\langle 1 \rangle \neg((\Box \Box A \rightarrow \Box A) \rightarrow \Box(\Box A \rightarrow A))$
2.  $\langle 1 \rangle \Box \Box A \rightarrow \Box A$  (1)
3.  $\langle 1 \rangle \neg \Box(\Box A \rightarrow A)$  (1)
4.  $\langle 1, 2 \rangle \neg(\Box A \rightarrow A)$  (3)
5.  $\langle 1, 2 \rangle \Box A$  (4)
6.  $\langle 1, 2 \rangle \neg A$  (4)
7.  $\langle 1 \rangle \neg \Box \Box A$  (2) | 8.  $\langle 1 \rangle \Box A$  (2)
9.  $\langle 1, 3 \rangle \neg \Box A$  (7) | 11.  $\langle 1, 2 \rangle A$  (8)
10.  $\langle 1, 3, 4 \rangle \neg A$  (9) | ⊗

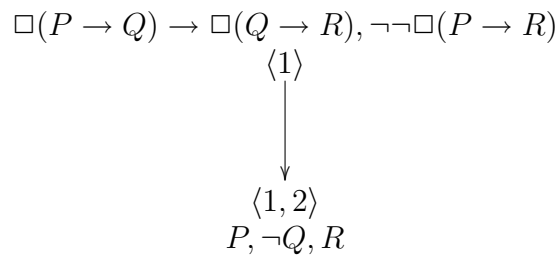
Ei **K**-pätevä.



3. a)

- |     |   |     |
|-----|---|-----|
| 1.  | $\langle 1 \rangle \neg \left( (\Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)) \rightarrow \neg\Box(P \rightarrow R) \right)$ |     |
| 2.  | $\langle 1 \rangle \Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)$   | (1) |
| 3.  | $\langle 1 \rangle \neg\neg\Box(P \rightarrow R)$   | (1) |
| 4.  | $\langle 1 \rangle \Box(P \rightarrow R)$   | (3) |
| 5.  | $\langle 1 \rangle \neg\Box(P \rightarrow Q)$   | (2) |
| 7.  | $\langle 1, 2 \rangle \neg(P \rightarrow Q)$  | (5) |
| 8.  | $\langle 1, 2 \rangle P$  | (7) |
| 9.  | $\langle 1, 2 \rangle \neg Q$   | (7) |
| 10. | $\langle 1, 2 \rangle P \rightarrow R$  | (4) |
| 11. | $\langle 1, 2 \rangle \neg P$ (10)  |     |
|     | 12. $\langle 1, 2 \rangle R$ (10)   | (2) |
- ⊗

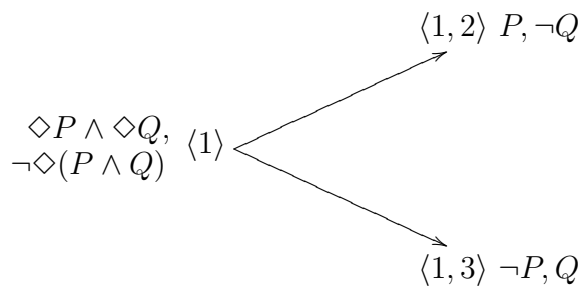
Ei **K**-pätevä.



b)

- |  |   |
|--|---|
| 1. $\langle 1 \rangle \neg((\diamond P \wedge \diamond Q) \rightarrow \diamond(P \wedge Q))$ |   |
| 2. $\langle 1 \rangle \diamond P \wedge \diamond Q$  | (1)   |
| 3. $\langle 1 \rangle \neg \diamond(P \wedge Q)$   | (1)   |
| 4. $\langle 1 \rangle \diamond P$  | (2)   |
| 5. $\langle 1 \rangle \diamond Q$  | (2)   |
| 6. $\langle 1 \rangle \Box \neg(P \wedge Q)$   | (3)   |
| 7. $\langle 1, 2 \rangle P$  | (4)   |
| 8. $\langle 1, 2 \rangle \neg(P \wedge Q)$   | (6)   |
| 9. $\langle 1, 2 \rangle \neg P$ (8)   | 10. $\langle 1, 2 \rangle \neg Q$ (8)   |
| $\otimes$  | 11. $\langle 1, 3 \rangle Q$ (5)  |
|  | 12. $\langle 1, 3 \rangle \neg(P \wedge Q)$ (6)                                 |
|  | 13. $\langle 1, 3 \rangle \neg P$ (11)   13. $\langle 1, 3 \rangle \neg Q$ (12) |
|  | $\otimes$   |

Ei **K**-pätevä.



c)

- |  |   |
|--|---|
| 1. $\langle 1 \rangle \neg(\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q))$ |   |
| 2. $\langle 1 \rangle \Box(P \wedge Q)$  | (1)                                       |
| 3. $\langle 1 \rangle \neg(\Box P \wedge \Box Q)$                                | (1)                                       |
| 4. $\langle 1 \rangle \neg \Box P$ (3)   | 5. $\langle 1 \rangle \neg \Box Q$ (3)    |
| 6. $\langle 1, 2 \rangle \neg P$ (4)   | 10. $\langle 1, 2 \rangle \neg Q$ (5)     |
| 7. $\langle 1, 2 \rangle P \wedge Q$ (2)   | 11. $\langle 1, 2 \rangle P \wedge Q$ (2) |
| 8. $\langle 1, 2 \rangle P$ (7)  | 12. $\langle 1, 2 \rangle P$ (11)         |
| 9. $\langle 1, 2 \rangle Q$ (7)  | 13. $\langle 1, 2 \rangle Q$ (11)         |
| $\otimes$  | $\otimes$                                 |