T-79.5101 **Spring 2008**
Advanced Course in Computational Logic
Exercise Session 11

1. Let $\mathcal{M} = \langle S, R, v\rangle$, where

$$\begin{aligned} S &= \{a, b, c, d, e, f\} \\ R &= \{\langle a, b\rangle, \langle b, c\rangle, \langle b, d\rangle, \langle b, e\rangle, \langle c, d\rangle, \langle d, d\rangle, \\ &\quad \langle e, e\rangle, \langle e, f\rangle, \langle f, a\rangle\} \\ \{s \in S \mid v(s, P) = \text{true}\} &= \{a, b, e\} \\ \{s \in S \mid v(s, Q) = \text{true}\} &= \{c, f\} \\ \{s \in S \mid v(s, R) = \text{true}\} &= \{f\} \end{aligned}$$

Let $F = \{R\}$. Which of the following claims hold?

a) $\mathcal{M}, a \models \mathbf{A}(P\mathbf{U}Q)$

b) $\mathcal{M}, a \models_F \mathbf{A}(P\mathbf{U}Q)$

c) $\mathcal{M}, a \models \mathbf{EG}P$

d) $\mathcal{M}, a \models_F \mathbf{EG}P$

2. Let $\mathcal{M} = \langle S, R, v\rangle$, where

$$\begin{aligned} S &= \{a, b, c, d, e\} \\ R &= \{\langle a, b\rangle, \langle a, c\rangle, \langle a, d\rangle, \langle b, c\rangle, \langle b, d\rangle, \langle c, a\rangle, \\ &\quad \langle c, e\rangle, \langle d, b\rangle, \langle d, e\rangle, \langle e, b\rangle\} \\ \{s \in S \mid v(s, P) = \text{true}\} &= \{a, b\} \\ \{s \in S \mid v(s, Q) = \text{true}\} &= \{b, c, d\} \end{aligned}$$

Give the states in which $\mathbf{AXE}\big((P \to Q)\mathbf{U}(P \wedge Q)\big)$ is true.

3. Let $\mathcal{M} = \langle S, R, v\rangle$, where

$$\begin{aligned} S &= \{a, b, c, d, e\} \\ R &= \{\langle a, b\rangle, \langle a, c\rangle, \langle b, d\rangle, \langle d, b\rangle, \langle c, e\rangle, \langle e, c\rangle, \\ &\quad \langle d, e\rangle\} \\ \{s \in S \mid v(s, P) = \text{true}\} &= \{a, c\} \\ \{s \in S \mid v(s, Q) = \text{true}\} &= \{b, c\} \end{aligned}$$

Give the states in which $\mathbf{AG}\big(Q \to \mathbf{A}(\mathbf{EF}P\mathbf{U}\mathbf{AF}P)\big)$ is true.

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Exercise Session 12

1. Let $\mathcal{M} = \langle S, R, v\rangle$, where

$$\begin{aligned} S &= \{a, b, c, d\} \\ R &= \big\{\langle a, a\rangle, \langle a, b\rangle, \langle a, c\rangle, \langle a, d\rangle, \langle b, a\rangle, \langle b, d\rangle, \\ &\quad \langle c, d\rangle, \langle d, a\rangle, \langle d, d\rangle\big\} \\ \{s \in S \mid v(s, P) = \text{true}\} &= \{b, d\} \\ \{s \in S \mid v(s, Q) = \text{true}\} &= \{b\} \end{aligned}$$

Using the tableau-based LTL model checking method, determine whether the following holds: $\mathcal{M}, a \models \mathbf{EX}(\neg P\mathbf{U}Q)$.

2. Let $\mathcal{M} = \langle S, R, v\rangle$, where

$$\begin{aligned} S &= \{a, b, c\} \\ R &= \big\{\langle a, a\rangle, \langle a, b\rangle, \langle a, c\rangle, \langle b, b\rangle, \langle b, c\rangle, \langle c, b\rangle, \\ &\quad \langle c, c\rangle\big\} \\ \{s \in S \mid v(s, P) = \text{true}\} &= \{b, c\} \end{aligned}$$

Using the tableau-based LTL model checking method, determine whether the following holds: $\mathcal{M}, a \models \mathbf{AFG}P$.

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Exercise Session 13

1. Using tableaux, show that the following CTL formula is valid.

$$\big(Q \vee \big(P \wedge \mathbf{AXA}(P\mathbf{U}Q)\big)\big) \to \mathbf{A}(P\mathbf{U}Q)$$

2. Using tableaux, determine whether the following LTL formula is satisfiable.

$$\mathbf{GF}P \to \mathbf{GF}\neg P$$