T-79.5101 **Spring 2008**
Advanced Course in Computational Logic
Exercise Session 4

1. Let P be an atomic formula (atomic proposition) and $\mathbf{L}$ the set of all frames. Show that the following does not hold.

$$\{\Box P \rightarrow \Diamond P\} \models_{\mathbf{L}} \{\Box\Box\neg P\} \Longrightarrow \Diamond\Box P$$

2. Let P be an atomic formula and $\mathbf{L}$ the set of all frames. Show that the following does not hold.

$$\{\Diamond P \vee \Diamond Q\} \models_{\mathbf{L}} \{\neg\Box P\} \Longrightarrow \Diamond Q$$

3. Show that the following holds for arbitrary $\mathbf{L}, \Sigma, \Upsilon, P, Q$:

If $\Sigma \models_{\mathbf{L}} \Upsilon \Longrightarrow P \wedge \Box P \wedge \Box\Box P \wedge \Box\Box\Box P \rightarrow Q$,
then $\Sigma \cup \{P\} \models_{\mathbf{L}} \Upsilon \Longrightarrow Q$.

4. a) Show that $\Box P \rightarrow \Box\Box P$ is valid in any transitive frame.

b) Show that $\neg\Box P \rightarrow \Box\neg\Box P$ is valid in any euclidean frame.

5. Show that any frame which is both reflexive and euclidean is also symmetric and transitive.

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Exercise Session 5

1. a) Show that $\Box P \rightarrow \Diamond P$ is valid in all serial frames.

b) Show that $\Box\Box P \rightarrow \Box P$ is valid in all weakly dense frames.

2. a) Give a Hilbert-style $\mathbf{K}$-proof for the formula $\Box P \rightarrow \Box(Q \rightarrow P)$.

b) Give a Hilbert-style $\mathbf{K}$-derivation for the formula

$$\Box\neg Q \rightarrow \Box\neg P$$

given the set of (global) premises $\{\Box(P \rightarrow Q)\}$. In other words, prove the following:

$$\{\Box(P \rightarrow Q)\} \vdash_{\mathbf{K}} \emptyset \Longrightarrow \Box\neg Q \rightarrow \Box\neg P.$$

3. a) Show that

$$\{P \rightarrow Q, \neg Q \rightarrow P, R\} \vdash_{\mathbf{K}} \{\neg R \vee Q, \neg Q \vee S\} \Longrightarrow \Box Q \wedge S$$

holds by giving a $\mathbf{K}$-derivation for the formula $\Box Q \wedge S$.

b) Show that the following holds.

$$\{\Diamond Q \rightarrow \Box Q, Q \rightarrow \neg P\} \vdash_{\mathbf{K}} \{\Diamond P\} \Longrightarrow \Box\neg Q.$$

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Exercise Session 6

1. Show that the following formulas are **K**-valid by providing a **K**-tableau proof for each formula.

a) $\Box P \rightarrow \Box(Q \rightarrow P)$

b) $\Box(P \rightarrow Q) \rightarrow (\neg\Box\neg P \rightarrow \neg\Box\neg Q)$

c) $(\Box P \wedge \Box Q) \rightarrow \Box(P \wedge Q)$

2. Are the following formulas **K**-valid? In each case: if not, give a counter-model, i.e., a model for the negation of the formula.

a) $\Diamond A \rightarrow \Box A$

b) $\Diamond\Box A \vee \Box\Diamond\neg A$

c) $(\Box\Box A \rightarrow \Box A) \rightarrow \Box(\Box A \rightarrow A)$

3. Are the following formulas **K**-valid? In each case: if not, give a model for the negation of the formula.

a) $(\Box(P \rightarrow Q) \rightarrow \Box(Q \rightarrow R)) \rightarrow \neg\Box(P \rightarrow R)$

b) $(\Diamond P \wedge \Diamond Q) \rightarrow \Diamond(P \wedge Q)$

c) $\Box(P \wedge Q) \rightarrow (\Box P \wedge \Box Q)$

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Exercise Session 7

1. Prove the following claims using tableaux.

a) $\Diamond(P \vee \Diamond\Box\Box P) \rightarrow \Diamond P$ is **S4**-valid (**S4** is the set of frames which are reflexive and transitive).

b) $\Diamond(\Diamond P \rightarrow \Box(\Diamond P \vee P))$ is **T**-valid (**T** is the set of reflexive frames).

c) $\Box(\Box(\Box P \wedge Q) \rightarrow \Diamond\Box\Diamond\Diamond(P \vee Q))$ is **KB**-valid (**KB** is the set of symmetric frames).

d) $\Box P \rightarrow \Diamond((P \rightarrow \Box Q) \rightarrow Q)$ is **D4**-valid (**D4** is the set of frames which are serial and transitive).

e) $\Diamond(\Box\Diamond\Box P \rightarrow \Box P)$ is **S5**-valid (**S5** is the set of frames which are reflexive, symmetric, and transitive).

2. Determine using tableaux whether $\Diamond P \rightarrow \Diamond\Box P$ is **K**-valid or **K4**-valid (**K** is the set of all frames and **K4** the set of transitive frames).

3. Show that $\Sigma \models_{\mathbf{K}} \{\neg P\} \Longrightarrow (\Diamond P \rightarrow \Diamond\Diamond P) \wedge \neg P$ holds, where

$$\Sigma = \{\Box P \rightarrow P, \Box P \rightarrow \Box\Box P, \Box\neg P \rightarrow \neg P, \Box\neg P \rightarrow \Box\Box\neg P\}.$$