T-79.5201 Discrete Structures, Autumn 2006

Tutorial 1, 27 September

1. Solve the following recurrence equations using the method of generating functions:

   (a)

   $$\begin{cases} a_0 = 0, \quad a_1 = 1, \\ a_n = 5a_{n-1} - 6a_{n-2}, \quad n \geq 2; \end{cases}$$

   (b)

   $$\begin{cases} b_0 = 0, \quad b_1 = 1, \\ b_n = 4b_{n-1} - 5b_{n-2}, \quad n \geq 2. \end{cases}$$

2. Let $\langle a_k \rangle = \langle a_0, a_1, a_2, \ldots \rangle$ be a sequence of real numbers, and $a(x)$ its real-valued generating function (i.e. the formal variable x is here also considered to be real-valued). Assume that the power series $\sum_{k \geq 0} a_k x^k$ converges in some neighbourhood of the origin. Which real-number sequences are then represented by the functions $a'(x)$ ja $\int_0^x a(t)\,dt$, defined in the same neighbourhood about the origin?

   Use these observations to determine the (real-valued) generating functions for the sequences $\langle 0, 1, 2, \ldots \rangle$ ja $\langle 1, \frac{1}{2}, \frac{1}{3}, \ldots \rangle$.

3. The *Stirling number of the second kind* $\left\{ n \atop k \right\}$ indicates in how many ways a set of n elements can be partitioned into k nonempty subsets. These numbers satisfy the recurrence equation

   $$\left\{ n \atop k \right\} = \left\{ n-1 \atop k-1 \right\} + k \left\{ n-1 \atop k \right\} \quad \text{when } (n,k) \neq (0,0); \qquad \left\{ 0 \atop 0 \right\} = 1.$$

   Using this recurrence, construct the generating function $S_k(z)$ for the sequence $\langle s_n \rangle$, where $s_n = \left\{ n \atop k \right\}$ (i.e. the sequence of Stirling numbers for a fixed value of k). Derive furthermore from the function $S_k(z)$ some estimates on the rate of growth of the numbers $\left\{ n \atop k \right\}$, as a function of n.