

T-79.5201 Discrete Structures, Autumn 2006

Tutorial 2, 4 October

1. Let $F(X) = \sum_{n \geq 0} a_n X^n$ be a formal power series. Prove the following results:
 - (a) $F' = 0$ if and only if $F = a_0 = \text{constant}$;
 - (b) $F' = F$ if and only if $F = a_0 \cdot \text{Exp}(X)$.
2. Let $F(X) = \sum_{n \geq 0} a_n X^n$, $G(X) = \sum_{m \geq 0} b_m X^m$ and $F_i(X) = \sum_{k \geq 0} c_{ik} X^k$ ($i = 0, 1, \dots$) be formal power series. Prove the following rules concerning the differentiation of products, infinite sums, and compositions of series:

$$\begin{aligned} D F(X)G(X) &= F'(X)G(X) + F(X)G'(X), \\ D \sum_{i \geq 0} F_i(X) &= \sum_{i \geq 0} F'_i(X), \\ D G(F(X)) &= G'(F(X)) \cdot F'(X). \end{aligned}$$

Which constraints on the coefficient sequences does one need to take into account when applying these rules?

3. Let $F(X) = \sum_{n \geq 1} a_n X^n$ and $G(X) = \sum_{m \geq 1} b_m X^m$ be formal power series that satisfy $a_0 = b_0 = 0$ ja $a_1, b_1 \neq 0$. Show that if $F(G(X)) = X$, then also $G(F(X)) = X$. (Thus for a given series F the “right” and “left” inverse series $G = F^{[-1]}$ coincide.) Determine by a formal calculation the three first coefficients of the series $\text{Ln}(1 + X) = (\text{Exp}(X) - 1)^{[-1]}$.