T-79.5201 Discrete Structures, Autumn 2006

Tutorial 9, 13 December

1. Use Darboux's lemma (Theorems 7.2 & 7.3 in the lecture notes) to estimate the coefficients of the following generating functions:

   (a) $f(z) = e^{-z/2}\sqrt{1-z}$,

   (b) $f(z) = e^{-z+z^2/2}\sqrt{1-z^2}$.

2. The exponential generating function for the class of *involutions* is $\hat{t}(z) = e^{z+z^2/2}$. (Cf. e.g. p. 28 of the lecture notes.) Use this fact to estimate the number t_n of involutions of n elements.

3. The exponential generating function of the Bell numbers, i.e. the numbers of partitions b_n of n-element sets is $\hat{b}(z) = \exp(e^z - 1)$. (Cf. e.g. tutorial 3 problem 2, or p. 25 of the lecture notes.) Use this fact to estimate the size of the numbers b_n.

In case you want to investigate the quality of your estimates in problems 2 and 3, you can easily obtain initial segments of the respective sequences from the "Online Encyclopedia of Integer Sequences" server, `http://www.research.att.com/~njas/sequences/`. Alternatively, you can determine the recurrence formulas for computing the exact values by applying the "$zD\log$ trick" to the egf's given in the problems, or recall them from previous exercises.