T-79.5201 Discrete Structures, Autumn 2007
Tutorial 2, 3 October

1. [Alon & Spencer, Prob. 2.3:] Prove that every set of n non-zero **real** numbers contains a subset A of **strictly** more than $n/3$ numbers such that there are no $a_1, a_2, a_3 \in A$ satisfying $a_1 + a_2 = a_3$.

2. Let σ be a permutation of $[n] = \{1, 2, \ldots, n\}$. Index i is a *left maximum* of σ if $\sigma(j) < \sigma(i)$ for all $j < i$. Compute the expected number of left maxima in a random permutation $\sigma \in S_n$.

3. ["Sperner's Theorem", Alon & Spencer, Prob. 2.7:] Let $\mathcal{F}$ be a family of subsets of $[n] = \{1, 2, \ldots, n\}$ and suppose there are no $A, B \in \mathcal{F}$ satisfying $A \subseteq B$. Prove that $|\mathcal{F}| \le \binom{n}{\lfloor n/2 \rfloor}$. (*Hint:* Let $\sigma \in S_n$ be a random permutation of $[n]$ and define the random variable

$$X = |\{i : \{\sigma(1), \sigma(2), \ldots, \sigma(i)\} \in \mathcal{F}\}|.$$

Consider the expectation of X.)