T-79.5201 Discrete Structures, Autumn 2007
Tutorial 7, 21 November

1. Show that the condition "$4pd \leq 1$" in the Lovász Local Lemma (Theorem 6.1) can be replaced by "$ep(d+1) \leq 1$". (*Hint:* Apply Theorem 6.2 with all $x_i = 1/(d+1)$ and use the estimate $1 - t \leq e^{-t}$.)

2. Let $\mathcal{F}$ be a k-uniform, k-regular hypergraph with $k \geq 10$. Show that $\mathcal{F}$ is 2-colourable.

3. The *van der Waerden number* $W(k)$ is the smallest n such that any colouring of the numbers $\{1, \ldots, n\}$ with two colours contains a monochromatic arithmetic progression with k terms. Show that $W(k) > 2^k/(2ek)$. (*Hint:* Assume that $n \leq 2^k/(2ek)$ and observe that one progression with k terms intersects at most nk others.)