S-72.2420/T-79.5203 Graph Theory (5 cr) Spring 2008 Programming Project

An $m \times n$ *grid graph* has a vertex set $V = \{(i, j) : 1 \leq i \leq m, 1 \leq j \leq n\}$ and two vertices, (i, j) and (i', j'), are adjacent if either $i = i'$ and $|j - j'| = 1$, or $j = j'$ and $|i - i'| = 1$. The 3×5 grid graph is depicted in Figure 1.

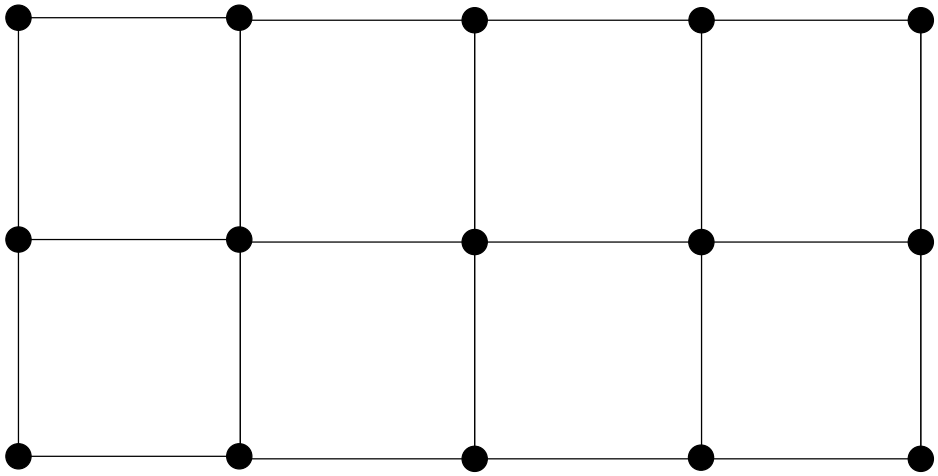

Figure 1: The 3×5 grid graph

The task in this programming project is to find the values of $D(m, n)$, the size of a minimum dominating set in an $m \times n$ grid graph, for small values of m and n by backtrack search. The results are to be documented in a report (in pdf, no more than 5 pages of text plus tables and possible figures); no source code should be appended to the report. The report must include a table of the bounds obtained on $D(m, n)$ and for each entry the amount of CPU time consumed. Also address the following questions in the report.

- Find an analytical lower bound on $D(m, n)$ (that is, a function of m and n). Perhaps you can also find some general upper bound on $D(m, n)$?

Freedom is given to what extent this problem is considered as well as to methods that are applied. The work is done in groups consisting of 1–3 students.

Some hints:

- The way in which the backtrack search proceeds has a profound impact on the speed of the algorithm.
- There is some symmetry in this problem; this can be utilized to get additional speed-up.
- The additional question above is closely related to a function that can be used to prune the backtrack search effectively.

The time scale for the project is as follows:

14.3. Presentation of problem, forming groups. Please register your group via e-mail to ⟨`olli.pottonen@tkk.fi`⟩. The e-mail should contain the names and student numbers of group members. If your group has fewer than 3 students, please indicate whether or not you want to be merged with another group.
21.3. Deadline for group registration.
25.4. Project review (oral presentation & written report).

The reports are subjected to peer review. The marking of your fellow students is in the formula for the mark of the course (see the general information of the course). The reports should be handed out during project review: one copy for each group and each of the two teachers. Each group is also to give a short oral presentation during the review describing the findings and the techniques employed.

Learning Objectives. Upon completion of the project, the students are able to

- develop basic graph algorithms;
- apply theoretical results in the development of algorithms;
- collaborate with fellow students in a science project;
- report project results concisely orally and in writing;
- assess algorithms and results produced by others.