T-79.5204 **Spring 2007**

Combinatorial Models and Stochastic Algorithms

Tutorial 2, February 1

Problems

1. Let $0 < p, q < 1$. Consider the Markov chain on state set $\{1, 2\}$ given by transition probability matrix

$$P = \begin{pmatrix} 1-p & p \\ q & 1-q \end{pmatrix}.$$

   Compute explicit expressions for the k-step transition probability matrices P^k, $k \geq 1$, and their (elementwise) limit $P^\infty = \lim_{k\to\infty} P^k$. Observe that if ρ is any distribution vector, then $\rho P^\infty = \pi$, where π is the stationary distribution of the chain.

2. Given the transition probability matrix P of an irreducible Markov chain on state set $S = \{1, \ldots, n\}$, show how to obtain from P the expected hitting times μ_{i1} for initial states $i \neq 1$. Use this result to compute the expected time to reach state 0 from each of the other states in a cyclic random walk on set $\{0, 1, 2, 3\}$ that at each step moves from state i to state $i \pm 1 \pmod 4$ with probability $1/2$.

3. Consider Problem 3(a) from the previous tutorial, i.e. the Markov chain determined by a king making random moves on a chessboard. You presumably already showed that this chain is irreducible and aperiodic, and hence has a unique stationary distribution π. Determine π. (*Hint:* View the chessboard as a graph.)