Problems to sections 2 and 3 of "Algebraic Graph Theory" by N. Biggs

1. A graph G is called bipartite if its vertex set can be split into two parts V_1 and V_2 such that every edge has one end in V_1 and another one in V_2. We denote a complete bipartite graph with $|V_1| = n$, $|V_2| = m$ by $K_{n,m}$ (every vertex in V_1 is connected with all the vertices in V_2). Compute the spectrum of $K_{n,m}$ (find all the eigenvalues and their multiplicities).

2. Let A be the adjacency matrix of a connected graph G with diameter d. Prove that d equals the minimum value of k such that all the entries of matrix $(I + A)^k$ are non-zero. Using this fact, suggest an efficient algorithm for computing diameters of connected graphs.

3. It is proved in the beginning of section 3 that the multiplicity of eigenvalue k of a k-regular connected graph is 1 (Proposition 3.1). What is the multiplicity of k for an arbitrary k-regular graph? In general, what can be said of the spectrum of a graph consisting of several connected components?

4. We denote the complete graph with $|V| = n$ by K_n. Prove that a graph obtained from K_7 by removing three arbitrary edges is not a line graph.