T-79.7001

Problems to sections 4 and 5 of "Algebraic Graph Theory" by N. Biggs

1. The transpose of the incidence matrix D^t is called the coboundary mapping. Show that the kernel of the coboundary mapping is a vector space with dimension c where c is the number of components of the graph. Show that a component of the graph is an element of this vector space and conclude that these provide a basis. Show that the image of the coboundary mapping is the cut-subspace.

2. Show that the cycle-subspace of a disconnected graph is the sum of cycle-subspaces of its component graphs. Show also that the cut-subspace of a disconnected graph is the sum of the cut-subspaces of its component graphs.

3. Consider the set of all connected graphs. Characterize a graph such that the cut-subspace is equal to the edge-space.

4. Let K_n denote the complete graph with n vertices. Consider the basis for the cycle-subspace obtained by the method of Theorem 5.2. Determine a spanning tree that produces as small cycles as possible as basis and a spanning tree that produces basis that contain one of the longest possible cycles. Determine also the corresponding basis for the cycle-subspace and the cut-subspace.