Problems to sections 6 and 7 of "Algebraic Graph Theory" by N.Biggs

1. *[page 42, problem* **6a***]*
If Γ is a connected k-regular graph with n vertices, show using Corollary 6.5 and the arithmetic-geometric mean inequality:
$$\kappa(\Gamma) \leq \frac{1}{n}\left(\frac{nk}{n-1}\right)^{n-1}$$
with equality if and only if $\Gamma = K_n$.

2. *[page 49, problem* **7b***]*
The characteristic polynomial of a tree: Suppose that $\sum c_i \lambda^{n-i}$ is the characteristic polynomial of a tree with n vertices. Show that the odd coefficients c_{2r+1} are zero, and the even coefficients c_{2r} are given by the rule that $(-1)^r c_{2r}$ is the number of ways of choosing r disjoint edges in the tree.

3. *[first part of page 49, problem* **7d***]*
The σ function of a star graph: A *star graph* is a complete bipartite graph $K_{1,b}$. For such a graph we can calculate σ explicitly from the formula of Theorem 7.5. Show that
$$\sigma(K_{1,b}\mu) = \mu(\mu - b - 1)(\mu - 1)^{b-1}$$

4. Let $K_{n,m}$ be the complete bipartite graph (cmp. problems to sections 2 and 3). Calculate the number of *elementary subgraphs* of $K_{n,m}$.