T-79.7001 **Spring 2006**
Postgraduate Course in Theoretical Computer Science
Problems, 3 April

1. For a given graph G, describe the relationship between the eigenvalues and eigenvectors of its normalised Laplacian $\mathcal{L}$ and its adjacency matrix A.

2. Verify the correctness of the characterisations (1.3) and (1.4) of the second (normalised) Laplacian eigenvalue of a graph, given on p. 5 of Chung's textbook.

3. Show that the second Laplacian eigenvalue of the complete graph K_n is $n/(n-1)$ using (one of) the variational characterisations given in equations (1.2)–(1.4) of Chung's textbook. [It follows then from Lemma 1.7 (ii) in Chung's book that in fact $n/(n-1)$ has multiplicity $n-1$, determining the complete spectrum of K_n.]

4. Compute the Laplacian spectrum of the complete bipartite graph $K_{m,n}$ using the techniques in Sections 1.2–1.3 of Chung's textbook. [*Caveat:* This is Example 1.2 on p. 6 of Chung's book, but as of this writing, I don't know how difficult it is to verify.]